Australian Government
The Treasury

TSY/AU

The Treasury Macroeconometric Model of Australia: Modelling Approach

Jared Bullen, Benjamin Conigrave, Adam Elderfield, Cecilia Karmel, Larissa Lucas, Chris Murphy, Heather Ruberl, Nicholas Stoney and Hui Yao[1]

Treasury Paper[2]

2021-09

Date created: September 2021

Date modified: September 2021

1 Macroeconomic Conditions Division, The Treasury, Langton Crescent, Parkes ACT 2600, Australia. Correspondence: Nicholas Stoney (nicholas.stoney@treasury.gov.au) and Jared Bullen (jared.bullen@treasury.gov.au).

2 The views expressed in this paper are those of the authors and do not necessarily reflect those of The Australian Treasury or the Australian Government.

The Treasury Macroeconometric Model of Australia: Modelling Approach.
Jared Bullen, Benjamin Conigrave, Adam Elderfield, Cecilia Karmel, Larissa Lucas, Chris Murphy, Heather Ruberl, Nicholas Stoney and Hui Yao
2021-09
3 September 2021

Abstract

Economy-wide models are an important tool used by fiscal authorities and central banks to support the provision of sound assessment of the economic outlook. The Treasury Macroeconometric Model of Australia ("EMMA") is a framework to support macroeconomic forecasting, and counterfactual policy and scenario analysis at the Treasury. It has been developed, tested and used within Treasury since 2018.

EMMA plays a central role in preparing the economic estimates which underpin Australian Government fiscal projections and scenario analysis which is used to assess the risks about those estimates. Macroeconomic forecasts and the scenario analysis from the model are also used to inform Treasury's advice across a broad spectrum of policy areas to assist with both policy formulation and policy costing. The model is also used to develop capability in applied small open economy macroeconomics.

The focus of this paper is to outline the model's structure and describe its place within the macroeconomic modelling literature. Further details on the model's equations, empirical properties and dynamics will be published over time. A macroeconomic model is never finished. The model will continue to be developed to ensure that it remains a fit for purpose tool to analyse the evolving economic and policy environment. This paper outlines the first version of the model.

JEL Classification Numbers: C32, C53, E10, E17, E27, E37, E47
Keywords: Australian economy, macroeconomic model.

Nick Stoney
Macroeconomic Division
Macroeconometric Modelling Unit
The Treasury
Langton Crescent
Parkes ACT 2600

Jared Bullen
Macroeconomic Division
Macroeconometric Modelling Unit
The Treasury
Langton Crescent
Parkes ACT 2600

Acknowledgements

The Treasury Macroeconometric Model of Australia has been developed in Treasury by Jared Bullen, Benjamin Conigrave, Adam Elderfield, Cecilia Karmel, Larissa Lucas, Heather Ruberl, Nicholas Stoney and Hui Yao. The development of the model was guided by macroeconomic modelling consultants Professor Mardi Dungey and Chris Murphy in advisory roles.

Professor Mardi Dungey was engaged as a macroeconomic modelling consultant from the start of the project. Mardi sadly passed away in January 2019. Mardi has worked closely with Treasury over many years and was a friend and a mentor to many within the Department. Mardi always showed great enthusiasm and intellectual rigour to problems, mentored and supported staff, and promoted top research, especially in empirical economics and finance. Mardi provided econometric training and used her expertise to assist in the development of the econometric framework of the model, including on the specification of key equations in the model.

Chris Murphy was also engaged as a macroeconomic modelling consultant for the project. Drawing on his extensive experience in macroeconomic modelling, Chris has made a major contribution to the development of the model in an advisory role, in particular in the development of the business and financial sectors of the model. He has also had the leading role in writing this paper.

We thank Professor Adrian Pagan and Professor Warwick McKibbin for their expert guidance through discussions during the model's early development. We also thank the macroeconomic modelling team at the Reserve Bank of Australia, and in particular, Daniel Rees, for sharing an early version of their macroeconometric model, MARTIN, and their experiences in building that model.

We would particularly like to thank Nigel Ray, Meghan Quinn and Angelia Grant for their support and guidance in the design of the model, along with the Macroeconometric Model Steering Committee (Nigel Ray, Meghan Quinn, Angelia Grant, Michael Kouparitsas, Jacqui Jones, Alex Heath and Daniel Rees) that oversaw the project to build the model.

We also thank Treasury's Expert Panel on Macroeconomic Modelling (Professor Heather Anderson, Dr Alex Heath, Dr Wendy Edelberg, Professor Warwick McKibbin, Professor James Morley, Professor Bruce Preston, Professor Kalvinder Shields, Dr Stephen Murchison, Dr Tim Robinson, Dr John McDermott and Associate Professor Chung Tran) for their guidance and continued and patient engagement with the project. And, we thank Peter Downes for his advice and assistance during the model's early development.

The macroeconometric model continues to be used and developed by the macroeconometric modelling unit in Treasury comprising Jared Bullen, Adam Elderfield, Hui Yao, Adam Hamilton, Duncan White and Jessica Xu.

The views expressed in this paper are those of the authors and do not necessarily reflect those of The Australian Treasury or the Australian Government. Any errors are our own.

CONTENTS

1. Introduction

The Treasury Macroeconometric Model of Australia ("EMMA") is a framework to support macroeconomic forecasting, and counterfactual policy and economic scenario analysis at the Treasury. EMMA plays a central role in preparing the economic estimates which underpin Australian Government fiscal projections. Macroeconomic forecasts and scenario analysis from the model are also used to inform and complement Treasury's advice across a broad spectrum of policy formulation and costings. The model is also used to develop capability in applied small open economy macroeconomics.

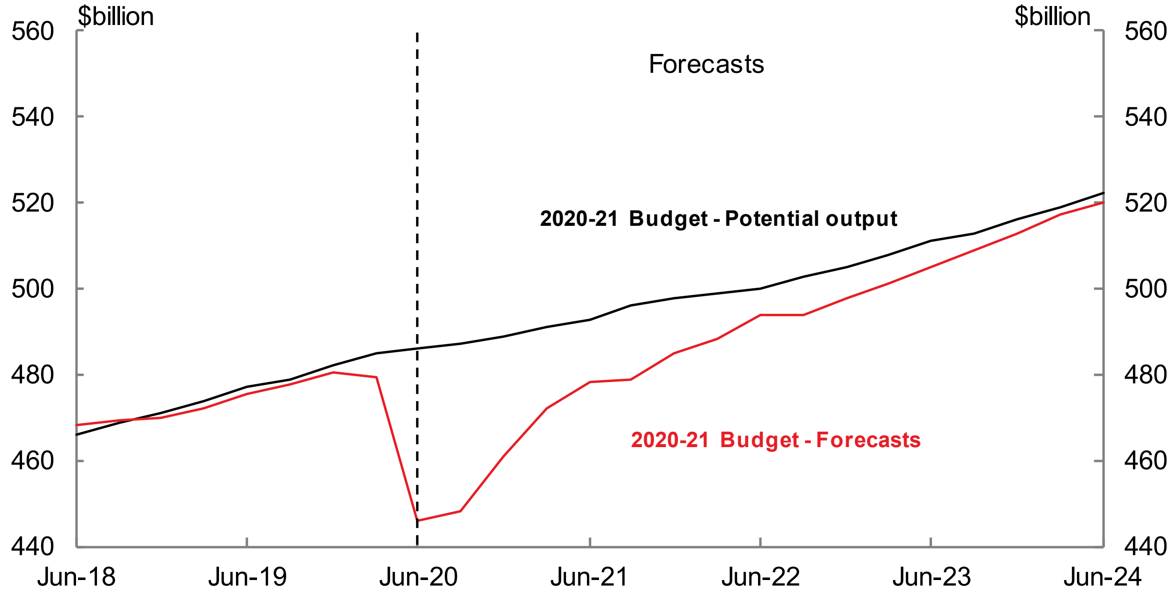
EMMA is part of a suite of models developed, maintained and used by the Treasury for assessing the economic outlook and analysing the effects of policy. In addition to EMMA, the Treasury has developed two macroeconomic models used specifically for counterfactual policy analysis — an overlapping generations model ("OLGA") used primarily for fiscal policy analysis and the Treasury Industry Model ("TIM") used for analysis that requires significant industry-level detail (see Box 1). These models have different advantages, but they have been designed to complement each other. At the Treasury, insights from different models and non-modelling approaches are combined to produce economic forecasts and assessments of the effects of specific policies.

Role of the macroeconometric model

The fiscal aggregates in the Budget are underpinned by forecasts of economic activity over the forward estimates and projections over the medium term.

In the forecasts for the Budget year and subsequent two financial years, greater emphasis is placed on detailed sectoral forecasts of the expenditure components of economic activity. Over this period, EMMA complements these detailed sectoral forecasts by aiding analysis of inter-connections between sectors of the economy.

Chart 1: Closure of the output gap in the 2020-21 Budget



Source: Treasury.

Beyond these detailed forecast years, estimates are constructed based on expectations for the level of potential output and modelling of the path by which output converges back to this potential level (Chart 1). Potential GDP is estimated within the macroeconometric model using exogenous inputs based on an analysis of trends for population, productivity, and participation. As spare capacity in the economy is absorbed over time (that is, the output gap closes), real GDP converges towards its potential level and the unemployment rate converges towards the estimate of the non-accelerating inflation rate of unemployment (NAIRU). On the nominal side, key non-rural commodity export prices are projected based on cost-curve analysis. Domestic prices return over time to the mid-point of the Reserve Bank of Australia's (RBA) inflation target band.

Over this period, EMMA plays a central role in informing the path that output takes to return to its potential level and domestic price inflation takes to return to the mid-point of the RBA's inflation target band.

The model is also used for sensitivity and scenario analysis, for example, scenarios modelled each year in the Budget Papers. This analysis typically involves changing one or more of the exogenous variables in the model such as mining commodity prices, trend productivity or global growth. The model is also used for evaluating alternative macroeconomic policies. EMMA incorporates a range of adjustment costs and frictions, which makes it well suited to analysing the short-run dynamic response of the economy to a shock or adjustment to policy settings. The model is therefore used to help to assess the short-run implications of fiscal policy.

These dual functions require striking a balance between the data consistency central to forecasting and the theory consistency important for projections and scenario analysis. In addition, the macro focus means that more attention needs to be paid to macro interactions than to industry, household or regional disaggregation.

History of macroeconometric models in Treasury

Treasury has a long history in developing and maintaining macroeconomic models of the Australian economy (Pagan, 2019).[3] Treasury has maintained a macroeconometric model since the 1970s. The first Treasury model, constructed in 1970, in conjunction with the Commonwealth Bureau of Census and Statistics (the forerunner of the Australian Bureau of Statistics), was the National Income Forecasting (NIF) model presented in Higgins (1970). The NIF model went through a considerable evolution, up to NIF-10, before a revamp as NIF-88; see Higgins and Fitzgerald (1973), Treasury (1981, 1984), and Simes and Horn (1988). NIF-88 was a medium-sized model, with 97 behavioural equations.

In 1991, the Treasury Macroeconomic Model (TRYM) replaced the NIF model (see Taplin et al., 1993). TRYM was smaller model (25 estimated behavioural equations), with more emphasis placed on the theoretical basis for equations and their steady-state properties. Most equations were specified in an error correction model format which made a clear distinction between short- and long-run properties (Hawkins, 2005). TRYM was used as a compliment to the National Accounts Forecasting Framework (NAFF), a spreadsheet system that built up forecasts of GDP from the components of the expenditure measure of GDP.[4] By the late 2000s, the model required redevelopment and gradually fell out use in the department.

The 2017 modelling review (Murphy, 2017) endorsed the recommendation of previous forecasting reviews that an economy-wide forecasting model should be developed and embedded in the wider forecasting process, and “that the new model be a macro-econometric model, combining strong short-term empirics with well-defined long-run properties.”

In response, the Treasury has built and tested EMMA alongside existing forecasting frameworks over a period of almost two years, principally 2018 and 2019, engaging regularly with their Expert Panel on Macroeconomic Modelling, the RBA, Professor Mardi Dungey and Chris Murphy. The model has been influenced by other models of the Australian economy, principally the model described in Murphy (2020), TRYM, and MARTIN, the RBA forecasting model (Ballantyne et. al. 2020) and also FRB/US, the US Federal Reserve forecasting model (Laforte, 2018). Beyond these general influences, the details of EMMA have been shaped by Treasury’s in-house research and Treasury’s own specific requirements for economic forecasting and scenario analysis.

Outline of the paper

This paper explains the general nature of EMMA by referring to just 20 key equations. The focus of this paper is to outline the model’s structure and describe its place within the macroeconomic modelling literature. Further details on the model’s equations, empirical properties and dynamics will be published over time.

The paper begins by discussing the general design of EMMA. This includes the choice of type of macro model and the approaches to economic theory, expectations, dynamics and estimation. It then considers the behaviour of each category of economic agent. Household, business, foreign and

3 Pagan (2019) describes the history of major macroeconometric models in Australia, with a particular focus on models constructed in Treasury and the Reserve Bank of Australia.

4 Forecasts for the expenditure measures of GDP draw upon structural econometrically estimated single equations, leading indicators, business liaison insights and expert judgement. National account identities are preserved within the spreadsheet, and consistency between forecast elements is achieved by iteration between sector specialists.

government behaviour are considered in turn. Finally, the interactions between economic agents in markets and the associated long-run economic outcomes are considered. More specifically, the workings of the product, labour and financial markets are discussed and the nature of the model's steady state path is explained.

Standard notation has been used for equations in the paper. The symbol Δ refers to the quarterly change in the value of a variable, * denotes the equilibrium value of a variable. A subscript i is used to refer to the three industries in the model: non-commodities, mining and agriculture. A subscript t refers to the time period.

2. General Design

As a macroeconomic model, EMMA focusses on macroeconomic interactions and is sparing with disaggregation. Hence it models a single, national economy with no state or other regional disaggregation. Similarly, there is a single, representative household making both labour supply and consumption decisions.

Three industries are distinguished – agriculture, mining and non-commodities. The separate treatment of mining is important for understanding fluctuations in business investment, while the separate treatments of agriculture and mining are useful in modelling exports. Indeed, distinguishing mining as a separate industry has become a feature of Australian macro models including in the dynamic stochastic general equilibrium (DSGE) model of Rees et al. (2016) at the RBA, AUS-M developed by Peter Downes and in the macroeconometric model of Murphy (2020).

Type of Model

The macroeconomy is highly complex, and hence constructing a workable model necessarily involves making simplifying assumptions. The types of simplifications that can be made, without significantly reducing the usefulness of a model, will depend on the model's intended purposes. Thus, the first step in developing a model such as EMMA is to identify its intended purposes.

Blanchard (2018) distinguishes five different purposes for macroeconomic models. Foundational models, such as the Mehra and Prescott model of the equity risk premium and the Mundell-Fleming model, make a theoretical point. DSGE models provide a platform for discussions of macroeconomic theory (although in practice they are used more widely than this, as discussed below). Policy models are used to model the dynamic effects of policy and other shocks. Toy models, such as the IS-LM (Investment-Saving - Liquidity Preference - Money Supply) model, are used as pedagogical devices in undergraduate textbooks. Forecasting models aim to give the best forecasts.

EMMA is used in Treasury for two of these five purposes: policy/scenario analysis and forecasting. Foundational models and toy models, alternatively referred to as academic miniature models by Fukač and Pagan (2011), are not designed for this. Rather, fiscal authorities such as the Treasury necessarily use macroeconomic models that are larger in scale for their detailed forecasting and scenario analysis work.

Fukač and Pagan (2011) discuss how these larger scale macroeconomic models have evolved over four generations, from 1G to 4G. 1G models of the 1950s and 1960s focussed mainly on aggregate demand, using an IS-LM framework. 2G models of the 1970s and 1980s introduced a production function to model aggregate supply. 3G models of the 1990s had an economically-interpretable steady state in which producers optimised profits, and also made some use of model consistent expectations.

In the 2000s, 4G models introduced intertemporal optimisation by households, replaced flexible dynamics with economically-interpretable dynamics and made shocks part of the model. These 4G models are more widely known as DSGE models. To some extent, the evolution from 1G to 4G models has reflected an increased emphasis on economic theory, perhaps indicating a shift in purpose from short-term forecasting to medium-term policy analysis.

Pagan (2019) describes how this evolution process played out in Australia. At Treasury, the NIF model evolved from the 1G NIF1 model through to the 2G NIF-88 model. Elsewhere Murphy et al. (1986) developed the 3G AMPS model that was further developed into the Murphy Model (Murphy, 1988) at the ANU. Treasury then developed TRYM (Taplin et al., 1993), a 3G model that was "close in structure to the Murphy model" but somewhat larger (Pagan, 2019). AUS-M further developed the TRYM model by incorporating industry disaggregation (Downes et al., 2014). Murphy (2020) has developed a new model from scratch incorporating several new features, but it remains unmistakably a 3G model.

In recent years, the RBA has developed two macroeconomic models for different purposes. First, it developed a 4G/DSGE model for policy analysis (Rees et al., 2016). Then, in keeping with the recommendations of the Pagan and Wilcox (2016) review of economic forecasts, it developed a separate macroeconometric model known as MARTIN for forecasting and scenario analysis (Ballantyne et al., 2020). MARTIN consists of a system of reduced form equations built to strike a balance between theoretical rigour and empirical realism. Most of the model's equations align closely with the way RBA staff typically interpret the behaviour of individual economic variables.

In comparing the strengths and weaknesses of these models, the RBA (Cusbert and Kendall, 2018) found as follows.

> *'DSGE models are built on a consistent theoretical framework of optimising households and firms, which provides a clear interpretation of the causal mechanisms within the model and an explicit role for forward-looking expectations. A weakness of DSGE models is that they often do not fit the data as well as other models, and the causal mechanisms do not always correspond to how economists and policymakers think the economy really works. In order to more easily manage these models, they typically focus on only a few key variables, which can limit the range of situations where they are useful.'*
>
> *'The key strength of … [a macroeconometric model] … is that it is flexible enough to incorporate the causal mechanisms that policymakers believe are important and fit the observable relationships in the data reasonably well. They can also be applied very broadly to model a wide range of variables. This flexibility reflects that the model is not derived from a single theoretical framework. However, this can make causal mechanisms less clear than in DSGE models. The model might capture an empirical relationship that exists in the data, but the cause of this might not be well understood. This means that developments may be more difficult to interpret and assumptions may need to be made about the mechanisms that are at work. If the true causal mechanisms are not correctly captured, empirical relationships may be unreliable over time or developments may be interpreted incorrectly.'*

In deciding what type of model EMMA should be (2G, 3G or 4G), the Treasury considered choices made by fiscal authorities in other countries. In the work for the Norwegian Ministry of Finance, Saxegaard (2017) surveyed the use of macroeconomic models in seven comparator countries - Sweden, Denmark, Finland, the Netherlands, Canada, the UK and New Zealand. He found as follows.

> *Most ministries of finance surveyed in this report used (or previously used) a large Macroeconometric model (LMM) as their primary tool for macro-fiscal forecasting and policy analysis. The exception is the Finnish Ministry of Finance which since 2011/12 has been using a small open economy Dynamic Stochastic General Equilibrium (DSGE) model.*

From Table 1 of the Saxegaard (2017) paper, the six LMM models in use in the comparator countries are a mixture of 2G and 3G models, while the Finnish DSGE model is a 4G model.

In his 2017 modelling review for the Treasury, Murphy considers four comparator countries – US, UK, Canada and New Zealand. He found that macroeconometric models play a substantial role in both macroeconomic forecasting and policy analysis in the comparator countries, with other types of models used for more specific purposes. In Murphy's review, the term 'macro-econometric models' referred to 2G and 3G models. Thus, the experience of these comparator countries seems to suggest that, for a fiscal authority such as The Treasury, a 2G/3G model is likely to provide wider benefits than a DSGE/4G model or a Vector Auto-Regression (VAR) model.

Macroeconometric models are the oldest and most eclectic of the three types of models. Their data consistency has been challenged by VAR models and their theory consistency by DSGE models. However, by striking an eclectic balance between data consistency and theory consistency, a macroeconometric model such as EMMA may be more suitable for the dual roles of forecasting and scenario analysis. This balanced approach in EMMA can be seen in its approaches to economic theory, agent expectations, dynamics and estimation, which are discussed in turn below. The various decisions made in these areas mean that EMMA looks more like a 3G model than a 2G or 4G model. While most of the academic literature focuses on DSGE models, a literature has emerged on the benefits of using more traditional structural econometric models for some applied tasks (see for example, Wren-Lewis (2018)).

Box 1. Treasury's suite of macroeconomic models

Different types of models have strengths and weaknesses, and some are better suited to particular tasks. In addition to EMMA, Treasury maintains a suite of modelling tools which it uses for policy analysis.

Treasury's workhorse general equilibrium model for fiscal policy analysis is an overlapping generations model (OLGA). OLGA is essentially a small open economy variant of the well-known lifecycle model developed by Auerbach and Kotlikoff (1987). It has been calibrated to Australian data and has a detailed description of the Australian tax and transfer system.

Treasury's principal model for industry policy analysis is a dynamic general equilibrium model of the Australian economy (TIM). At its core TIM is a small open economy version of the well-recognised neoclassical growth model known as the Ramsey Cass Koopmans model. In contrast to typical neoclassical growth models, TIM incorporates 114 forward looking, infinitely-lived firms that represent Australian industries. TIM includes significant detail on the linkages between industries and on industry-specific tax settings.

Due to the forward-looking behaviour of households and firms in these models they play an important role in analysing responses to policy where expectations about the future are important. However, Treasury's versions of these models are deterministic rather than stochastic and are therefore less suited to assessing business cycle dynamics and forecasting.

Economic Theory

EMMA draws on economic theory up to the point where it is considered realistic in modelling macroeconomic fluctuations. This involves using economic theory more for equilibrium relationships, which are discussed here, than for dynamics, which are discussed below.

Economic theory is used widely in deriving the long run relationships in EMMA. In the long run, a representative business in each industry maximises profit subject to a production function with primary factors of labour, capital and a fixed factor. In this long run equilibrium, all markets clear. Household and government intertemporal budget constraints are also enforced.

Economic theory is used in a more pragmatic way in modelling household behaviour in EMMA. Decisions about labour supply, aggregate consumption, and the pattern of consumer demand across industries are modelled independently drawing on general economic principles, rather than in a unified way as part of an overarching process of intertemporal utility maximisation, as in DSGE models.

Agent Expectations

EMMA is pragmatic in making assumptions about whether economic agents have rational expectations. This places it between DSGE models, where all agents are assumed to have rational expectations, and VAR models, where agent expectations can be interpreted as backward looking. The reduced form solution to a linear rational expectations model is a VAR. However, such solutions will change if the policy regime changes (Lucas critique). Thus, if expectations are rational, a structural rational expectations model is needed to evaluate alternative policy regimes.

Expectations are assumed to be model consistent or rational[5] in EMMA's financial markets. This provides an operationally simple way of capturing the obvious forward-looking behaviour observed in these markets: asset prices jump when there is new information. Here, model-consistent expectations are not viewed as a literal portrayal of agent expectations, but rather as a convenient approximation.

In the labour market of EMMA, wage movements are influenced by household inflation expectations. However, households are assumed to be less sophisticated than financial markets in forming their expectations for the future. Rather than being model consistent, in EMMA household inflation expectations depend on a weighted average of backward-looking expectations (recent actual inflation) and forward-looking expectations (the RBA's inflation target).

In the product markets of EMMA's three industries, price movements are influenced by business inflation expectations. Like households, businesses are assumed to form their expectations in a simpler way than financial markets. Producer inflation expectations depend on the rate of inflation in the marginal cost of production and the RBA's inflation target.

Dynamics

As previously noted, consistency with the historical macroeconomic data is important for EMMA because of its use in forecasting. This emphasis on data consistency is seen in the approach taken to dynamics.

5 More precisely, in EMMA financial market expectations are model consistent in the sense that they are obtained as deterministic predictions from the model. As EMMA is mildly non-linear, these deterministic predictions may differ slightly from mean predictions from stochastic simulations. Also, no claim is made that EMMA uses all available information. For both reasons, it could be said that these model consistent expectations may not be strictly rational expectations, although some authors use these two terms interchangeably.

Dynamics are introduced flexibly using first order error correction models (ECMs). Many behavioural equations, including 10 out of the 20 key equations considered in this paper,[6] use an ECM(1) structure for dynamics that is either similar or the same as the following.

$$\Delta\log(y_t) = \beta_1[\log(y_{t-1}) - \log(y^*_{t-1})] + \beta_2\Delta\log(y^*_t) + (1-\beta_2)g^*_t$$

Under this dynamic structure, the growth rate in the endogenous variable, y, is modelled as a weighted average of the growth rates in its equilibrium value, y^* (denoted with y^* in all equations), and trend growth in the economy, g^*. The coefficient restriction on the response to changes in trend growth in the economy is imposed to avoid steady-state bias.[7] More importantly, y also adjusts to gradually close the outstanding disequilibrium (or 'error') of y from y^*. This overall design ensures that the equation reaches a steady state in which y is equal to $y^*$and both variables grow at the same rate.

This 2-parameter approach to dynamics (β_1 and β_2 above) is more flexible, and hence places more weight on the data, than the typical single parameter dynamics derived from optimisation, as generally used in DSGE models. For example, here price stickiness is modelled directly using an ECM(1) rather than from an underlying theory that assumes either that prices are costly to adjust or only a proportion of agents are free to optimise their prices in any given period.

Equally, this ECM(1) approach places less weight on the data than the general-to-specific approach of beginning with a higher order ECM and progressively testing restrictions. This is to reduce the risk of over-fitting that comes with higher order ECMs, for instance from capturing spurious correlations and over-specifying dynamics (see the discussion below).

Just like for the dynamic parameters, the general aim is to freely estimate the equilibrium parameters. For example, in the above ECM equation, the equilibrium value for y might be determined as follows.

$$\log(y^*_t) = \gamma_0 + \gamma_1\log(x_t) + \gamma_2\log(z_t)$$

In EMMA, this expression for y^* is generally substituted into the dynamic equation prior to estimation, so that both the equilibrium and dynamic parameters are estimated in the following single regression.

$$\Delta\log(y_t) = \beta_1[\log(y_{t-1}) - \gamma_0 - \gamma_1\log(x_{t-1}) - \gamma_2\log(z_{t-1})]$$
$$+ \beta_2\Delta[\gamma_1\log(x_t) + \gamma_2\log(z_t)] + (1-\beta_2)g^*_t$$

In this regression, the values for γ are generally freely estimated in the first instance. However, depending on the outcome of the free estimation, some γ values may be imposed if this is necessary for consistency with the empirical literature and/or for obtaining plausible model simulation properties.

6 Equations 2, 5, 6, 8, 9, 10, 11, 12, 13 and 14 follow this form.

7 That is, this parameter restrictions ensures that if there is no disequilibrium error in the previous period t-1 and if y* and y both grow at their steady-state rate of growth in period t, then y will equal its equilibrium value y* in period t.

Estimation

The emphasis on data consistency for forecasting is also seen in the approach taken to estimation. The behavioural equations in the model are estimated using quarterly data extending as far back as 1980. However, for some equations shorter estimation periods are used to avoid suspected structural changes in the 1980s or 1990s.

The behavioural equations are estimated using single equation regressions estimated through maximum likelihood. This approach places more weight on the data than imposing values on behavioural parameters or using Bayesian estimation with informative priors.

Data consistency is a reason that single equation estimation is used in preference to systems estimation. However, single equation estimation risks over-specifying model dynamics, for instance with correlations between macroeconomic variables captured both in the single equation and through the model's dynamic solution. To reduce this risk, as discussed above, parsimony in the inclusion of dynamic terms has been prioritised over maximising model fit in the single equation estimation.

An alternative approach to address this issue is system estimation. The much larger number of parameters being estimated at the same time under systems estimation means more reliance needs to be placed on imposed values or Bayesian estimation. Fukač and Pagan (2010) show how Bayesian full information, a popular estimator of DSGE models, can lead researchers to unknowingly adopt parameter values that lie outside of confidence intervals that would be generated if Bayesian priors were not used.

3. Approach to households

As noted above, household decisions about labour supply, aggregate consumption and consumer demand are modelled independently. The approach to these household decisions is discussed in turn.

Labour supply

EMMA models labour input in total hours worked. Thus, labour input can vary both at the extensive margin (heads) and the intensive margin (average hours worked). In modelling labour supply, the labour force participation rate equation is formulated on a heads basis. There is no supply equation for average hours worked. Rather, average hours are assumed to be determined by demand on an industry-by-industry basis, as discussed under the approach to businesses.

The modelling of the labour force participation rate begins with a trend labour force participation rate. This trend rate is determined outside of the model by aggregating trend participation rates for different cohorts. The methodology for modelling the trend labour force participation rate is outlined in Gustafsson (2021). EMMA then models, at the aggregate level, cyclical variations in the participation rate around this trend, based on cyclical variations in employment reflecting an encouraged worker effect.

Equation 1: Labour force participation rate (cyclical component)

$$\Delta \log(\rho_t) = \beta_1[\log(\rho_{t-1}) - \log(\rho^*_{t-1})] + \beta_2\left[\log\left(\frac{N_t}{POP_t}\right) - \log\left(\frac{N^*_t}{POP^*_t}\right)\right] + \beta_3 \Delta \log\left(\frac{N_{t-1}}{POP_{t-1}}\right) \quad (1)$$

Where: ρ_t is the participation rate, N_t is heads employment and POP_t is population.

Thus, under this approach, the participation rate is affected by demographics, social changes captured within the trend labour force participation rate model, and the encouraged worker effect. During the projection period it is not affected by real after-tax wages, so EMMA is not designed for assessing how a change in the tax burden on labour may influence labour supply.

Aggregate consumption

Household consumption (C) is modelled using a modern, empirical version of the Ando-Modigliani (A-M) consumption equation adapted from Aron et al. (2012). The framework is relatively undemanding in the sophistication required of households (Aron et al. (2012)). That is, households are assumed to have only a basic understanding of a life-cycle budget constraint, in contrast to the assumed 'well-informed households' of alternative models of household consumption – see below. The A-M consumption function fits the data reasonably well, allows a role for counter-cyclical fiscal policy and has desirable long-run properties, in particular it is consistent with long run balanced growth in the model, which are important for EMMA's forecasting and policy analysis purposes.

In the model, household decision-making is inconsistent with Ricardian equivalence as households are assumed to have static expectations. Hence, the model displays standard short-run Keynesian properties in response to a debt-financed tax cut; that is, household consumption and in turn aggregate demand are stimulated by a tax cut.

EMMA models equilibrium consumption as a linear homogenous function of current non-property income (Y^{NP}), housing wealth[8] (VW^{H}) and non-housing wealth (VW^{NH}) (equation 2a). Under this approach, non-property income influences consumption directly, while property income appears in capitalised form as wealth, which is valued at replacement cost. This is a departure from Aron et al. (2012), where equilibrium consumption is linear homogeneous in current and permanent non-property income and wealth. The inclusion of permanent non-property income may be considered in future model development work.

The EMMA consumption equation includes some refinements that are also from Aron et al. (2012). Equilibrium consumption depends negatively on the real cash interest rate. Further, actual consumption adjusts to equilibrium consumption in an error correction model (ECM). Finally, income uncertainty, as measured by the increase in the unemployment rate, has a transitory, negative effect (equation 2).

Equation 2: Household consumption

$$\Delta\log(C_t) = \beta_6[\log(C_{t-1}) - \log(C^*_{t-1})] + \beta_7\Delta\log(Y_t^{NP}) + (1-\beta_7)\Delta\log(Y_t^*) + \beta_8\Delta U_t \quad (2)$$

Equilibrium household consumption

$$\log(C_t^*) = \beta_1 + \beta_2 dum\_c_t + \log(Y_t^{NP}) + \beta_3\left(\frac{VW_{t-1}^{H}}{Y_t^{NP}}\right) + \beta_4\left(\frac{VW_{t-1}^{NH}}{Y_t^{NP}}\right) + \beta_5 R_t \quad (2a)$$

Where: C is real consumption, Y^{NP} is real non-property income, ΔY^* is potential output growth, ΔU is the change in the unemployment rate, dum_c_t is a dummy that has a value of one until 2007 Q2 and zero afterwards, VW^{H} is real housing wealth, VW^{NH} is real non-housing wealth and R is the real cash rate, calculated as the nominal cash rate deflated using the change in through-the-year trimmed mean inflation.

A feature of the Ando-Modigliani consumption function is that households implicitly target a ratio of net wealth-to-income in the long run for given rates of return on assets and the steady-state growth rate of after-tax labour income. The long-run target ratio of net wealth-to-income is shown below in a stylised way, abstracting from differences in rates of return on assets, differences in propensities to consume out of different categories of wealth and from returns on the fixed factors of production.

$$\varpi = \frac{1-\omega}{\phi - (r-g)}\,(W\,NH - T^N)$$

Where: ϖ is private wealth, ω is the marginal propensity to consume out of income, ϕ is the marginal propensity to consume out of wealth, r is the rate of return on assets, g is the growth rate of the economy, W is the nominal wage, NH is total hours worked and T^N is labour income tax.

The target implicitly reflects the household's consumption, saving and leisure preferences over the estimation sample period. Under two conditions ($\phi > r - g; 0 < \omega < 1$), consumption is a positive proportion of income, where the proportion lies between zero and unity. Under these conditions, in the long run, household consumption is proportional to after-tax labour income. Hence, in the long run, consumption and GDP grow at the same rate and there can be balanced growth in

8 Housing wealth includes structures and land. Modelling the value of the stock of housing land is complex and is the subject of ongoing research.

GDP and consumption. In the long-run, household wealth is also proportional to after-tax labour income.

$$C = \frac{\phi - \omega(r-g)}{\phi - (r-g)} \left(W\ NH - T^N\right)$$

Box 2. Long run properties of Ando-Modigliani consumption equation

Some of the stylised long run implications of the A-M consumption equation can be seen by combining the consumption equation with the long run household income constraint, abstracting from differences in rates of return on assets, differences in propensities to consume out of categories of wealth and from returns on fixed factors of production.

The long run household income constraint is obtained by using the long run national income and government budget constraints to eliminate government spending, G.

$$C + G - T^C = W\ NH + T^K + (r-g)\ (\varpi + B) \text{ (National Income Constraint)}$$

$$G + (r-g)\ B = T^K + T^C + T^N \text{ (Government Budget Constraint)}$$

Where: r is the rate of return on assets, g is the growth rate of the economy, W is the nominal wage, NH is total hours worked, T^N is labour income tax, T^K is tax on capital, T^C is tax on consumption, ϖ is private wealth, and B is government net debt.

The long run household income constraint shows that, in the steady state, household consumption is funded from after-tax labour income plus asset income net of sustainable saving.

$$C = W\ NH - T^N + r\ (\varpi + B) - g\ \varpi \text{ (Household Income Constraint))}$$

The household income constraint can be used to eliminate private wealth, ϖ, from the A-M consumption function to obtain the following reduced form equation for household consumption in the steady state.

$$C = \frac{\phi - \omega(r-g)}{\phi - (r-g)} \left(W\ NH - T^N\right)$$

Where: ω is the marginal propensity to consume out of current income, ϕ is the marginal propensity to consume out of wealth.

The associated reduced form equation for private wealth, ϖ, is as follows.

$$\varpi = \frac{1-\omega}{\phi - (r-g)} \left(W\ NH - T^N\right)$$

Some other Australian macroeconometric models follow a broadly similar approach to modelling household consumption. Specifically, in the RBA's MARTIN model (Ballantyne et al., 2020), in the model of Murphy (2020) and in EMMA, equilibrium household consumption is linear homogeneous in current income and household wealth. The common use of current income in modelling consumption means that a debt-financed tax cut stimulates household consumption and aggregate demand in the short run in all three models. At the same time, there are some more subtle differences between the consumption equations of the three models.

In MARTIN, current income refers to all of household income, whereas in EMMA it includes only the non-property component. EMMA follows Aron et al. (2012) who show that the influence of the property component of income on consumption can be captured indirectly through the inclusion of wealth. The two consumption equations also use different functional forms.

In Murphy (2020), the long-run target for wealth refers to national wealth, whereas household wealth is used in EMMA. Empirical testing is unlikely to be very useful in deciding between these subtly different long-run targets, leaving conceptual appeal as the more likely deciding factor. On the one hand, Murphy (2020) shows that his national wealth target has the appealing property of being consistent with long run fiscal neutrality. On the other hand, the use of household wealth does not require that households are eventually sophisticated enough to understand that government debt is a household liability. On balance, EMMA assumes that households target household wealth.

The approach to modelling household consumption is a key and noteworthy point of differentiation of EMMA and the two other Australian macroeconometric models from a DSGE model. The use of an A-M consumption function is common in macroeconometric models, but DSGE models typically use the Euler equation from the Ramsey model. In its basic form, this is derived from intertemporal utility maximisation for a representative consumer who behaves as a dynasty and forms expectations of the future based on the model. DSGE models are built on a consistent theoretical framework of optimising households and firms, which provides a clear interpretation of the causal mechanisms in the model and an explicit role for forward-looking expectations.

The assumption of fully optimising households in the Ramsey Euler equation is conceptually appealing. However, Murphy (2020) notes that it has two main drawbacks. First, unlike in the A-M approach, in the basic Euler equation there is no link from current after-tax income to consumption, making counter-cyclical fiscal policy implausibly ineffective. Second, in an open economy where the cost of capital, r, is determined abroad, the rate of growth in consumption from the Euler equation will usually not match the rate of growth in GDP in the long run, that is, growth is unbalanced. Long-run balanced growth can be enforced by imposing a constraint on the rate of time preference, known as a knife-edge condition. Other solutions for achieving long-run balanced growth are discussed from a theoretical perspective in Turnovsky (2002) and an econometric perspective in Schmitt-Grohé and Uríbe (2003).

Coenen et al. (2012) discuss how seven DSGE models in use at government authorities modify the basic Euler equation approach to generate more realistic dynamics in the relationship between consumption and current after-tax income. These modifications incorporate additional theory of household behaviour and deviations from fully rational forward-looking households. They include the introduction of hand-to-mouth consumers not subject to the Euler equation, a reduced elasticity of intertemporal substitution, allowing for habit persistence in consumption, and loosening the dependence of the local cost of capital on the cost abroad. These modifications arguably come at the cost of reduced clarity of the causal links in the model and additional demands on model estimation.

These issues with the fully optimising approach of the Ramsey Euler equation are partly addressed by Blanchard (1985) in his overlapping generations model (OLG). In his model, instead of behaving as a dynasty, the representative consumer has no concern for future generations, introducing a negative wealth effect on the rate of growth in consumption. The Blanchard OLG model replaces the knife-edge condition on the rate of time preference with a less restrictive condition often referred to in the OLG literature as the medium-term impatience condition. Under this condition, households consume a proportion out of their wealth, such that assets don't accumulate explosively. The model also introduces a link from current income to consumption. However, empirical evidence indicates this link is considerably stronger in practice.

For forecasting and short-term policy and scenario analysis, it is important to realistically model the link from fluctuations in current after-tax income to fluctuations in consumption. And, it is appealing to require of households a less demanding framework from which to base their consumption decisions, partly as the framework is more intuitive to policymakers. The A-M consumption function has these properties within a relatively parsimonious framework, noting this comes at the cost that the causal mechanisms are less clear than in a fully optimising approach, such as in the Ramsey or Blanchard OLG models.

Consumer demand

The total value of consumption determined in EMMA's A-M consumption equation is in turn allocated across four different consumer goods and services. These are supplied by EMMA's three industries – agriculture, mining and non-commodities – and its quasi-industry of housing services or ownership of dwellings.

Housing services are singled out for special treatment in modelling consumer demand in EMMA. This is because in the short-term the quantity of housing services is determined by the available supply, whereas the quantities of the other three consumer goods and services can be assumed to be demand determined. The market for housing services clears by the rental price gradually adjusting to match demand with the available supply.

At present, the three other goods and services are assumed to be consumed in fixed proportions. However, allowing for price-sensitive substitution between them is a possible area for future model development work.

4. Approach to businesses

The business sector describes the behaviour of firms in demanding primary factor inputs and intermediate inputs into production, combining domestic production with imports to construct total supply, and then supplying outputs to the domestic and export markets.

Firms decision making is driven by a desire to maximise profits. These decisions determine the productive output of the economy. In EMMA, a unified technology structure (shown in Figure 1) is used to model all of the firm's decisions, both in demanding inputs to production and in supplying output to markets. As a consequence, in the long run, a single profit maximisation problem explains the behaviour of firms and ensures internal consistency between the firm's input demand and supply decisions. It also ensures that factor inputs are paid their marginal product and that the zero pure profit condition is met. That is, all first order conditions necessary to maximise profits are satisfied.

All firm behaviour converges to a long-run equilibrium in which profit is maximised. In the mining and agriculture sectors, the adjustment process is driven by the costs of adjusting capital stocks. In the non-commodities sector, nominal price rigidities are also important drivers of the path to equilibrium, in addition to capital stock adjustment. Firms do not jointly optimise across all of their decisions along the transition path to long-run equilibrium.

The production and trade technology and the approach to these business decisions is discussed in turn.

Production and trade technology

In EMMA, the modelling of business behaviour is based around a production and trade technology that is shown in Figure 1. A similar technology is used for each of the three industries – agriculture, mining and non-commodities. The representative firm in each industry simultaneously makes decisions across four stages of the process of the production and distribution of output.

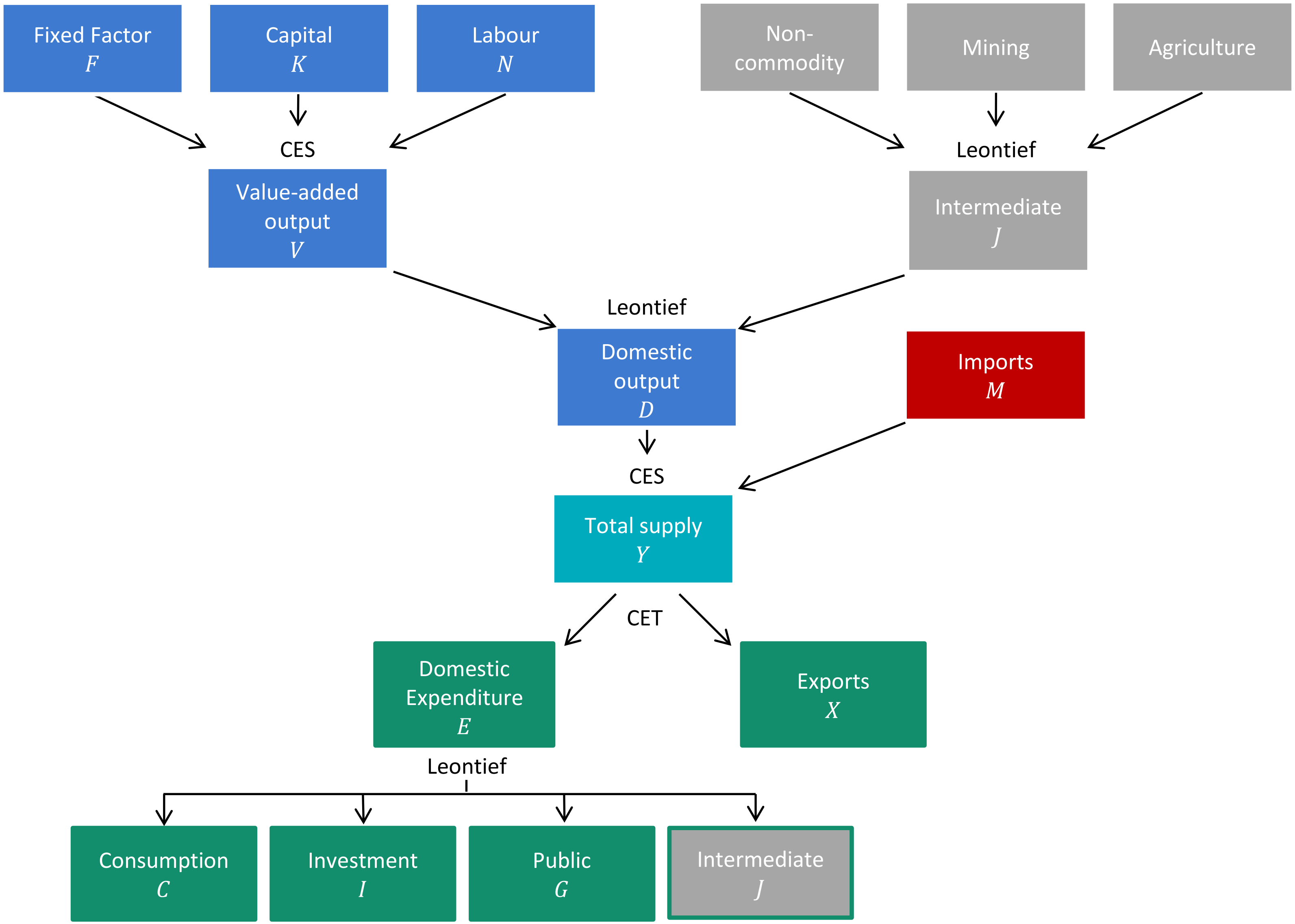
At the centre of the technology, the firm uses a production function in which the primary factors of capital (K), labour (N) and a fixed factor (F) are substitutable inputs in producing value-added output (V) (Stage 1). Labour input is measured in hours worked. The firm then combines value-added production with intermediate inputs to produce domestic production (D) (Stage 2).

The firm also acts as a distribution agent by combining domestic production with imports (M), which determines the firm's total supply (Y) (Stage 3), and then deciding how much of total supply will be allocated to the domestic (E) and export (X) markets (Stage 4).

EMMA's modelling of the production of domestic output (Stages 1 and 2) includes two refinements to the standard production technology.

The first refinement is that EMMA extends the usual production function involving primary factors of capital and labour to allow for an industry-specific fixed factor. This fixed factor is included in the production function for each industry, other than for the non-commodities industry, which is the largest industry in EMMA. The economic interpretation of the fixed factor varies with the industry. In agriculture it represents agricultural land, in mining it represents mineral resources, and in the quasi-industry of housing services, it represents housing land.

Figure 1: Production and trade technology



In each of these industries, it is useful to allow for the fixed factor for two reasons. First, the presence of a fixed factor reduces the flexibility of an industry's supply and taking this into account makes the model more realistic. Second, the fixed factor receives a significant share of industry income and ignoring that would mean the return to capital would be over-stated, leading to difficult-to-justify excessive industry risk premia.

The second refinement is that, as a multi-industry model, EMMA includes intermediate inputs as another factor of production. All three industries use intermediate inputs produced by all three industries. The inclusion of intermediate inputs allows for a deeper understanding of the inter-connections between industries and the flow-on implications of industry level shocks.

Regarding functional form, in the first stage of the production process the primary factors of capital, labour and, where applicable, a fixed factor, are combined in a Constant Elasticity of Substitution (CES) production function to produce value added output. The elasticity of substitution differs between industries but are estimated to be nearer to 0.5 than to the Leontief case of zero or the Cobb-Douglas case of unity. Along the balanced growth path all productivity growth is labour-augmenting. Further details on productivity are included in Appendix C.

In the second stage of the production process, value added output is combined with the intermediate inputs to produce domestic output. In this stage, Leontief technology is assumed. That is, intermediate inputs are combined in fixed proportions, and the resulting bundle of intermediates is then combined in fixed proportions with value added output.

This assumption of fixed proportions for intermediate inputs seems reasonable given that EMMA's three industries are very broad. It becomes more important to allow for substitutability between intermediates in models with finer industry disaggregation. Examples of this include models that distinguish different forms of energy, for example the G-cubed model of McKibbin and Wilcoxen (1999), or different modes of transport.

As mentioned, EMMA uses a unified technology structure to model both trade and production (Figure 1). As a consequence, in the long run, a single profit maximisation problem leads to industry decisions about import demand and export supply that are fully consistent with industry decisions about inputs to production. Regarding functional form, in the third stage of the production and distribution process, imports (M) are substitutable with domestic output (D) in generating total supply (Y) according to a CES function. In the fourth stage, that supply is transformable in meeting demands for exports (X) and domestic expenditure (E) according to a Constant Elasticity of Transformation (CET) function.

Trade in EMMA is modelled under the plausible assumption that Australia approximates a small open economy. Australia is assumed to be a price taker for imports. Australia is almost a price taker for exports from agriculture and mining, with high price elasticities, while in the remaining industry of non-commodities, export demand is also price elastic, but less so.

These high export demand elasticities would be likely to lead to implausibly volatile export volumes if export supply were similarly price elastic. Export supply in each industry would be highly price elastic in the long run under the following set of assumptions: constant returns to scale in production, all factors of production are variable, perfect competition, perfect labour mobility between industries and frictionless switching of industry supply between the domestic and export markets. Under those assumptions, each industry's export supply curve will be close to horizontal, with the supply price equal to the largely given unit cost of production.

To overcome this potential problem of volatile export volumes when highly price elastic export demand is combined with highly price elastic export supply, EMMA introduces two frictions in export supply. The first friction is the presence of the fixed factor of production in the agriculture and mining industries, which was mentioned above. These fixed factors impart upward slopes on the export supply curves for these two industries.

Following Powell and Gruen (1968), the second friction is that there is less than perfect transformability between supplying the domestic and export markets. As a consequence, a CET function is used in allocating total supply between exports and local expenditure. This means that there is an increasing opportunity cost in an industry switching supply from the domestic market to the export market, implying an upward sloping export supply curve.

Convergence to equilibrium

EMMA bases its long-run modelling of producer behaviour on standard economic theory. That is, in each of the three industries, in the long-run equilibrium a representative business maximises profit subject to the production and trade technology shown in Figure 1. At this stage the simple assumption is made that, in the long run, profit is maximised under perfect competition. Allowing for imperfect competition is under consideration as part of future model development.

Firms simultaneously make inter-dependent decisions about the level of production, the price to charge, and the use of variable, intermediate and fixed inputs to production in order to maximise profit. Firms do not jointly optimise across all of their decisions along the transition path to long-run equilibrium. Over time, variables converge to their equilibrium value, resulting in a convergence toward a long-run equilibrium across all decision variables. The first order conditions have been arranged to reflect stylistic features of the adjustment process for prices, volumes and factor inputs to their equilibrium values.

In the non-commodity industry, a hierarchical adjustment approach is employed in using the first order conditions for profit maximisation to drive the economy from the Keynesian short run to the Classical long run. In the Keynesian short run, equilibrium employment demand is obtained by inverting the production function. A sticky expenditure price then gradually adjusts to an equilibrium value based on short-run marginal cost. Once this equilibrium value is reached, the marginal product of labour condition is satisfied, so there is a medium-run equilibrium.

Investment demand is based on Tobin's-q theory of investment so the capital stock is costly to adjust. Once the capital stock adjustment process is complete, the q-ratio equals unity implying that the zero pure profit condition is satisfied. In this Classical long run, all profit maximising first-order conditions are met: the firm operates on its production function and the marginal product of labour and zero pure profit conditions are satisfied.

The dynamic adjustment process in the non-commodity industry is broadly speaking driven by two main economic elements: sticky prices and sluggish adjustment of capital stocks based on adjustment costs. The assumption of New Keynesian 'sticky prices' leads output to be demand determined in the short run. These nominal price rigidities are important in explaining short-run disequilibrium and provide a role for monetary and fiscal policy in managing aggregate demand.

In contrast, the adjustment process in the two commodities industries – agriculture and mining – is somewhat simpler. For reasons explained below, prices are assumed to be flexible so there is no Keynesian short run. Hence, the adjustment process is driven only by the costs of adjusting capital stocks.

Table 1: Stylised hierarchical adjustment process

The hierarchical adjustment for the non-commodities industry involves price adjustment in the medium run and capital adjustment in the long run. This description is stylised. Of course, some adjustment occurs in both variables in every quarter.[9]

	Non-Commodity	Commodity (Mining/Agriculture)
Short run (0-2 years)	• Output is demand determined • Firms gradually adjust labour input in response to short-run demand • Firms adjust factor utilisation resulting in temporary changes in Total Factor Productivity to meet remaining short-run demand • Capital stock fixed • Output prices are sticky	• Output is supply determined • Firms gradually adjust output in response to changes in input and output prices • Factor utilisation (and thus Total Factor Productivity) and labour input converge to equilibrium levels • Capital stock fixed • Output prices are flexible
Medium run (2-10 years)	• Output is supply determined • Factor utilisation reaches its equilibrium level • Labour inputs fully adjusted • Capital stock partially adjusts • Output prices are flexible and converge to marginal costs of production	• Output is supply determined • Factor utilisation reaches its equilibrium level • Labour inputs are fully employed • Capital stock partially adjusts • Output prices are flexible and converge to marginal costs of production
Long run (>10 years)	• Output is supply determined • Factor utilisation at equilibrium levels • Labour inputs fully adjusted • Capital stock fully adjusts • Output prices equal marginal costs • Zero pure profit	• Output is supply determined • Factor utilisation at equilibrium level • Labour inputs fully adjusted • Capital stock fully adjusts • Output prices equal marginal costs • Zero pure profit

9 Technically the short run of the model consists only in the initial shock quarter t. Beyond this quarter, prices and the capital stock start their gradual adjustment and so the non-commodities industry is not demand determined. The adjustment process has been presented in a stylised way to aid intuition.

In any case, in all three industries EMMA converges to a long-run equilibrium in which profit is maximised. There is sluggish adjustment of capital stocks in all industries and sticky prices in the non-commodities industry. These aspects of equilibrium behaviour and economic dynamics in the business sector of EMMA are now explained.

Business Investment

As noted above, the main factor in macroeconomic models slowing the adjustment of businesses to long-run equilibrium is the sluggish adjustment of capital stocks. To model this, EMMA adopts Tobin's-q theory of investment in which a representative firm in each industry maximises the present value of its after-tax cash flow. This occurs in the presence of investment adjustment costs that are used to explain the sluggish adjustment of capital.

The early literature assumes that these adjustment costs depend on the level of investment, while some more recent literature assumes they depend on the change in investment. The approach used here is to begin by assuming that adjustment costs depend on the level of investment before considering the alternative assumption later.

Investment adjustment costs are often described, somewhat vaguely, as installation and planning costs. In practice, whether these adjustment costs are interpreted as installation or planning costs can affect how they are modelled.

McKibbin and Wilcoxen (1999) interpret adjustment costs as installation costs. Installation costs would be incurred on all investment, including normal levels of investment. Hence McKibbin and Wilcoxen model investment adjustment costs, IA, as depending on total investment, I. This means that adjustment costs are even incurred in the steady state.

$$IA_t = \frac{\psi}{2}\frac{{I_t}^2}{K_t}$$

In contrast, Kudrna and Woodland (2011) appear to interpret adjustment costs as planning costs. Planning costs might only be incurred when the economy is not growing predictably on a steady state path. Hence, Kudrna and Woodland assume that adjustment costs only arise when investment deviates from its steady state level, $(\delta + gr)K$.

$$IA_t = \frac{\psi}{2}\frac{[I_t - (\delta + gr)K_t]^2}{K_t}$$

EMMA follows the Kudrna and Woodland formulation for adjustment costs. This means that there are no adjustment costs in the steady state.

Once an investment equation is estimated, the value for the adjustment cost parameter, ψ, can be recovered and used to derive a measure of adjustment costs using an equation like that for IA above. In some models, those derived adjustment costs are subtracted from investment before calculating changes in the capital stock. They can also be treated as a deductible expense for corporate tax. However, the slow capital stock adjustment speeds typically obtained when estimating Tobin's-q investment equations imply that adjustment costs are quite large, which raises a doubt about whether they should be taken literally.[10] For that reason, in EMMA, investment

10 For example, in practice slow capital adjustment speeds can be due partly to factors besides adjustment costs such as time-to-build and government approval processes. Attributing low speeds entirely to adjustment costs is therefore likely to result in those costs being overstated.

adjustment costs are only used as a device in deriving the investment equations and beyond that they are not used in the model.

The Kudrna and Woodland (2011) formulation for adjustment costs leads to an investment equation that takes the form shown below. The optimal rate of investment covers normal investment plus a disequilibrium component that depends on the incentive to invest as measured by the Tobin-q ratio. The disequilibrium component is positive or negative depending on whether the expected market value of an additional unit of capital, μ, exceeds or falls short of the replacement cost, *PI*.

When equilibrium is achieved in the long run, the market value of an additional unit of capital exactly equals its replacement cost. Equivalently, the zero pure profits condition of profit maximisation under perfect competition is met.

$$\frac{I_t}{K_t} = \delta + gr + \frac{1}{\psi}\left[\frac{\mu_{t+1,t}}{PI_t} - 1\right]$$

Three issues need to be considered in implementing this equation.

The first implementation issue is the treatment of expectations. Expectations for the market value of capital can be assumed to be either model-consistent (rational) or static. EMMA assumes these expectations are static. Assuming model-consistent expectations would require that investors are sophisticated enough to understand that a high q-ratio is likely to be gradually self-correcting, because the positive investment response will increase the supply of capital thus pushing the q-ratio back down. Understanding this process of self-correction would dampen the initial investment response for given adjustment costs.

The second implementation issue is the measurement of the market value of an additional unit of capital. One way of measuring this is using stock market prices. However, stock market prices are volatile and at times may not be driven by fundamentals, reducing their reliability as an indicator of investment incentives. An alternative option available under static expectations is to measure the market value of capital by capitalising current cash flow using the cost of capital. There is intuitive appeal in relating investment to cash flow. Given these considerations, in EMMA the market value of a unit of capital is constructed from current cash flow (and the cost of capital).

The third implementation issue is the point raised earlier about whether adjustment costs depend on the level or change in investment. While the traditional Tobin-q approach assumes that adjustment costs depend on the level of investment, Groth and Kahn (2010) find that the change in investment is also an important driver of adjustment costs. Indeed, the influential DSGE models of Christiano et al. (2005) and Smets and Wouters (2007) assume that adjustment costs depend entirely on the change in investment, rather than the level of investment, as does the RBA DSGE model of Rees et al. (2016). Interestingly, if adjustment costs depend on the change in investment, they are not incurred in the steady state. As already shown, this is also the case under the Kudrna and Woodland formulation of adjustment costs used in EMMA, although for different reasons.

While the EMMA investment equation is derived using the Kudrna and Woodland approach to adjustment costs, the idea that some adjustment costs depend on the change in investment is then introduced informally by specifying that investment depends on its lagged value. In practice, this considerably improves the fit of the investment equations.

These three implementation decisions are incorporated in the EMMA investment equations. The typical investment equation is fully derived at Appendix B. The use of the lagged dependent variable can be seen in the main equation (3), the normal level of investment is used as a benchmark in equation (3a) and current cash flow drives the actual rate of return in equation (3b) that is used in calculating Tobin-q. The cost of capital is modelled using the real 10-year bond rate, adjusted for risk in equation (3c).

Equation 3: Business investment rate in industry i (disequilibrium component)

$$IC_{i,t} = \beta_1 \left(AR_{i,t-1} - CC_{i,t-1}\right) + \beta_2 IC_{i,t-1} \tag{3}$$

Definition of disequilibrium component of business investment rate

$$IC_{i,t} \equiv \frac{I_{i,t}}{K_{i,t-1}} - \left(\Delta \log(Y_t^*) + \frac{\delta_{i,t}}{400}\right) \tag{3a}$$

Actual rate of return on capital from cash flow

$$AR_{i,t} = \frac{(1-\tau_{i,t}^C)\left[P_{i,t}^V \cdot V_{i,t} - \left(W_{i,t} \cdot NH_{i,t} + r_{i,t}^F \cdot F_{i,t}\right)\right]}{P_{i,t}^I \cdot K_{i,t}} + \tau_{i,t}^C \frac{\delta_{i,t}}{400} \tag{3b}$$

Cost of capital

$$CC_{i,t} = \frac{(i_t^{10} - \pi_t^e)}{400} + \left(1 + \frac{\pi_t^e}{400}\right)\frac{\delta_{i,t}}{400} + Risk_{i,t} \tag{3c}$$

Where: I is the level of investment, K is the capital stock, δ is the annual rate of depreciation, τ^C is the corporate tax rate, P^V is the price received for value-added output, V is value-added output, W is the hourly wage, NH is employment (in hours), r^F is the rental price of the fixed factor, F is the fixed factor input, P^I is the investment price deflator, i^{10} is the nominal 10 year bond rate, π^e is inflation expectations and $Risk_i$ is a risk premium.

Housing Investment

The same conceptual approach is used in modelling housing investment, as can be seen in equation 4. This means that the incentive to invest in housing depends positively on housing rents relative to construction costs and negatively on the real 10-year bond rate.

There are some implementation differences in modelling housing investment compared to business investment. The modelling is simpler for housing investment in that labour is not an input in producing housing services, and corporate tax is assumed not to apply. On the other hand, the modelling of housing investment allows for a more direct link from monetary policy via a negative effect from the difference between the cash rate and the 10-year bond rate (equation 4).

Equation 4: Housing investment rate (disequilibrium component)

$$IC_{odw,t} = \beta_1\left(AR_{odw,t} - CC_{odw,t}\right) + \beta_2 IC_{odw,t-1} - \beta_3\Delta_4\left(i_{t-1} - i_{t-1}^{10}\right) \tag{4}$$

Definition of disequilibrium component of housing investment rate

$$IC_{odw,t} \equiv \frac{I_{odw,t}}{K_{odw,t-1}} - \left(\Delta y_t^* + \frac{\delta_{odw,t}}{400}\right) \tag{4a}$$

Actual rate of return on capital

$$AR_{odw,t} = \frac{P_{odw,t}^V \cdot V_{odw,t} - r_{odw,t}^F \cdot F_{odw,t}}{P_{odw,t}^I \cdot K_{odw,t-1}} \tag{4b}$$

Cost of capital

$$CC_{odw,t} = \frac{\left(i_t^{10} - \pi_t^e\right)}{400} + \left(1 + \frac{\pi_t^e}{400}\right)\frac{\delta_{odw,t}}{400} + Risk_{odw,t} \tag{4c}$$

Where: i is the nominal cash rate.

Sticky vs flexible prices

Besides sluggish adjustment of capital, the other main economic factor in macroeconomic models slowing the adjustment of businesses to long-run equilibrium is stickiness in prices. However, prices are likely to adjust more rapidly than capital, so in EMMA price adjustment is modelled taking capital stocks as given. One can think of a hierarchy in which prices adjust in the medium run and capital in the long run, although of course some adjustment occurs in both variables in every quarter.

In modelling price adjustment, EMMA makes a distinction between its two commodity industries, agriculture and mining, and its non-commodity industry. The two commodity industries are assumed to be Classical, so that prices are flexible and producers operate on their supply curves. The non-commodity industry is assumed to be Keynesian, so that its price is sticky and output is demand determined in the short run.

The motivation for this distinction is that the two commodity industries are highly trade-oriented, so that their selling prices may be determined flexibly on world markets. In contrast, the more domestically-focussed non-commodity industry may choose to adjust prices slowly in the face of price adjustment costs. In the medium run all prices become fully flexible, so this distinction between the industries disappears. In the long run, capital stocks are also fully flexible.

This modelling approach of hierarchical adjustment with a distinction between Keynesian and Classical behaviour is broadly similar to Murphy (2020). One difference is that Murphy distinguishes between Keynesian domestic markets and Classical export markets, whereas EMMA distinguishes between Keynesian industries and Classical industries because this simplifies estimation.

This overview paper focusses mainly on the modelling of producer behaviour in the non-commodity industry where prices are sticky, as the non-commodity industry is considerably larger than the aggregate of the two commodity industries. Equilibrium equations of both the commodity and non-commodity sectors are included in Appendix D.

Price of local sales

Equilibrium industry prices in EMMA are derived from the supply-side of the economy, reflecting the cost of production at each stage of production. This approach to modelling prices from industry supply costs has important advantages in modelling the pass-through of production costs into output prices. A limitation of this approach is that, given the model currently only has three industries (plus the quasi-industry of housing services), it has limited ability to capture relative price movements between expenditure components that source industry final-use products in similar shares.

Sticky prices in the non-commodities industry are introduced through a gradual adjustment of prices to their equilibrium level. Expenditure price deflators are mapped from industry total supply prices, with weights reflecting the industry's share of total supply in the expenditure category. For example, the household consumption deflator (the model's proxy for the consumer price index) reflects a weighted average of the total supply price of the non-commodities, mining, agriculture and housing services industries, with weights reflecting industry shares of total supply in the national accounts household consumption bundle.

In modelling sticky prices in EMMA's non-commodity industry, there are three alternative points on the distribution chain that could be used. There is the price of domestic production, P^D, the price of total supply (which also includes imports), P^Y, and finally there is the price of sales on the local market (which excludes exports), P^E (see Figure 1). Sticky prices are modelled at the final point of local sales because that is the point where the costs of price adjustment may be greatest.

Sticky prices are modelled by first determining an equilibrium price of local sales, P^{E^*}, and then using an ECM to model the gradual adjustment of the actual price to that equilibrium price.

Modelling the equilibrium price of local sales involves working along the distribution chain, starting with the equilibrium price of value added, P^{V^*}. It is determined by the marginal cost of producing value added which varies positively with the wage rate and the output-capital ratio in equation (5a). Importantly, once the price adjusts to equal marginal cost, the marginal product of labour equals the real wage, a key condition for profit maximisation.

Next, the equilibrium price of domestic production, P^{D^*}, is determined as a weighted average of the equilibrium price of value added and the price of intermediates in equation (5b). At the next point in the distribution chain, the equilibrium price of total supply, P^{Y^*}, is determined in a CES cost function in equation (5c) by combining the equilibrium price of domestic production with the price of imports, P^M.

Finally, the equilibrium price of local sales is determined residually from the CET revenue function of equation (5d) in which the equilibrium price of total supply reflects the equilibrium prices received for local sales and exports, P^{X^*}. The actual price of local sales then adjusts to its equilibrium price in the ECM of equation (5).

Stickiness in the price of local sales has implications for the way the model responds to domestic demand shocks. In a Keynesian short run, the price stickiness means that a positive demand shock will be accommodated by higher domestic production, that is, production is demand determined.

This higher production will raise marginal cost, leading to gradual upward adjustment of price. In a Classical medium run, price has adjusted fully to marginal cost. This brings the marginal product of labour back into line with the real wage and producers revert to operating on their supply curves.

Equation 5: Price of local sales of industry in the non-commodities industry

$$\Delta\log(P^{E}_{NC,t}) = \beta_1\left[\log(P^{E}_{NC,t-1}) - \log(P^{E^*}_{i,t-1})\right] + \beta_2\Delta\log(P^{M}_{NC,t-1}) + \beta_3\Delta\log(P^{E^*}_{NC,t}) + (1-\beta_2-\beta_3)\pi^T_t \tag{5}$$

Equilibrium price of local sales, P^{E^*}, of industry *i* is determined recursively in a series of four equations as follows.

Equilibrium price of gross value added, P^{V^*}, is equal to its marginal cost of production, satisfying the marginal product of labour condition.

$$P^{V^*}_{NC,t} = \left(\frac{W_t}{\lambda^N_{NC,t}}\right)\left[\frac{1-\theta^{K\,\frac{1}{\sigma^V_i}}_{NC}\left(\frac{K_{i,t-1}}{V^*_{i,t}}\right)^{\frac{\sigma^V_i-1}{\sigma^V_i}}}{\theta^N_{NC}}\right]^{\frac{1}{\sigma^V_i-1}} \tag{5a}$$

Equilibrium price of domestic production, P^{D^*}, is then determined as a weighted average of the equilibrium price of gross value added, P^{V^*} and intermediate input prices.

$$P^{D^*}_{i,t} = \beta_v\, P^{V^*}_{i,t} + \sum_j \beta_{j,i}P^E_{j,t} \tag{5b}$$

Equilibrium price of total supply, P^{Y^*}, is then determined as a CES cost function in the equilibrium price of domestic production, P^{D^*}, and import prices, P^M.

$$P^{Y^*}_{i,t} = \left[\alpha_i P^{D^*\,1-\sigma^Y_i}_{i,t} + (1-\alpha_i)P^{M\,1-\sigma^Y_i}_{i,t}\right]^{\frac{1}{1-\sigma^Y_i}} \tag{5c}$$

Finally, the equilibrium price of local sales, P^{E^*}, is determined residually from a CET revenue function in which the equilibrium price of total supply, P^{Y^*}, reflects the equilibrium price of local sales, P^E, and export prices, P^X.

$$P^{Y^*}_{i,t} = \left[\phi_i P^{X^*\,1+\sigma^T_i}_{i,t} + (1-\phi_i)P^{E^*\,1+\sigma^T_i}_{i,t}\right]^{\frac{1}{1+\sigma^T_i}} \tag{5d}$$

Where: π^T is the mid-point of the RBA's inflation target band, λ^N is labour augmenting technical change, θ^K is capital's share of output, θ^N is labour's share of output, σ^V is the elasticity of substitution between factors of production, β_v is input share of value-added in domestic production, $\beta_{j,i}$ is the input share of intermediate inputs from industry j to domestic production in industry i, σ^Y is the elasticity of substitution between domestic production and imports in production total supply and σ^T is the elasticity of transformation in transforming domestic sales into exports.

Hours worked

The modelling of producer behaviour around the main production function is completed by the modelling of labour demand. As noted above, labour demand is measured using total hours worked. The equilibrium level of total hours worked, NH^*, is obtained by simply inverting the main production function, as seen in equation (6a). Actual total hours worked, NH, then adjusts to equilibrium using the ECM of equation (6).

Equation 6: Total hours worked in the non-commodities industry

$$\Delta\log(NH_{NC,t}) = \beta_1\left[(NH_{NC,t-1}) - \log(NH^*_{NC,t-1})\right] + \beta_2\Delta\log(NH^*_{NC,t}) + (1-\beta_2)(\Delta\log(POP^*_t) + \Delta\log(H^*_t) + \Delta\log(\rho^*_t)) \quad (6)$$

Equilibrium total hours worked (by inverting this CES production function)

$$V^*_{NC,t} = A_{NC,t}\left[{\theta^N_{NC}}^{\frac{1}{\sigma^V_{NC}}}\left(\lambda^N_{NC,t}NH^*_{NC,t}\right)^{\frac{\sigma^V_{NC}-1}{\sigma^V_{NC}}} + {\theta^K_{NC}}^{\frac{1}{\sigma^V_{NC}}}K_{NC,t-1}{}^{\frac{\sigma^V_{NC}-1}{\sigma^V_{NC}}}\right]^{\frac{\sigma^V_{NC}}{\sigma^V_{NC}-1}} \quad (6a)$$

Where: POP^* is trend population, NH^* is trend average hours worked and ρ^* is trend labour force participation

This inverted production function approach links labour demand to output. Thus, both output and employment are demand determined in a Keynesian short run but producers operate on their supply curves in a Classical medium run once prices have fully adjusted.

In the long run, the business sector meets the conditions to maximise profit in relation to the main production function through the combined effect of business investment, key price and labour demand equations. These ensure that there is zero pure profit, labour is paid its marginal product and the business operates on its production function respectively.

Because labour input is measured using total hours worked, it is necessary to decompose this into employment on a heads basis, which is used in modelling unemployment, and average hours worked. Trend average hours worked is exogenous to the model and is based on an aggregation of average hours worked projections across age gender cohorts. Average hours worked are modelled to vary pro-cyclically with total hours worked in equation (7). Employment on a heads basis is then obtained by dividing total hours worked by average hours worked.

Equation 7: Average hours worked in industry i

$$\log(H^c_{i,t}) = \beta_1\Delta\log(NH^c_{i,t}) + \beta_2\log(H^c_{i,t-1}) \quad (7)$$

Cyclical average hours worked in industry i

$$\log(H^c_{i,t}) \equiv \log(H_{i,t}) - \log(H^*_{i,t}) \quad (7a)$$

Cyclical total hours worked in industry i

$$\log(NH^c_{i,t}) \equiv \log(NH_{i,t}) - \log(NH^*_{i,t}) \quad (7b)$$

Import demand and export supply

In EMMA, trade flows are integrated with the production technology. This means producer behaviour determines demand for imports and supply of exports.

The modelling of producer behaviour and trade flows is based around total supply/demand. In each industry, producers meet demand for local sales and for exports by transforming goods and services for each market using a CET function. This total demand is met using a CES combination of local production and competing imports.

In equilibrium in each industry, the representative producer chooses the combination of local production and competing imports that minimises cost and the combination of local sales and exports that maximises revenue. The focus here is on how that operates in the non-commodities industry.

Cost minimisation determines the equilibrium import propensity in equation (8a). This import propensity depends negatively on the price of imports. Actual imports then adjust to equilibrium imports, M^*, in the ECM of equation (8).

Equation 8: Imports that compete with industry i

$$\Delta\log(\mathrm{M}) = \beta_1[\log(M_{i,t-1}) - \log(M^*_{i,t-1})] + \beta_2\Delta\log(Y_{i,t}) + \beta_3\left(\Delta\log\left(P^M_{i,t}\right) - \Delta\log\left(P^Y_{i,t}\right)\right) + (1-\beta_2)\Delta\log(Y^*_t) \tag{8}$$

Equilibrium imports in competition with industry i (based on cost minimising combination of imports, M, and domestic production, D, in producing total supply, Y)

$$M^*_{i,t} = (1-\alpha_i)Y_{i,t}\left(\frac{P^{Y\,*}_{i,t}}{P^M_{i,t}}\right)^{\sigma^Y_i} \tag{8a}$$

This approach models a single import propensity for each industry that does not vary between different end uses. An alternative approach would be to allow for different import propensities for, say, an industry's intermediate goods, investment goods and consumption goods. That disaggregation may have advantages in the industry policy setting of a computable general equilibrium (CGE) model. However, the EMMA approach of disaggregating imports by industry but not by end use seems suitable for its purpose of analysing macroeconomic fluctuations.

Under revenue maximisation, the optimal ratio of exports to local sales depends positively on their relative price. However, in the non-commodities industry, the representative producer does not directly control these quantities because they are demand determined in the Keynesian short run. Instead, the producer gradually adjusts the sticky export price until export demand is matched to the revenue-maximising export supply.

The equilibrium supply price of exports is modelled in equation (9a). The actual price of exports then adjusts to the equilibrium price in the ECM of equation (9).

Equation 9: Price of exports for the non-commodities industry

$$\Delta\log(P^X_{NC,t}) = \beta_1\big(\log(P^X_{NC,t-1}) - \beta_2 P^X_{NC,t} trend - \log(P^{X*}_{NC,t-1})\big) + \beta_3\left(\Delta\log\left(\frac{P^W_t}{\varepsilon_t}\right)\right) + (1-\beta_3)\Delta\log({P^X_{NC,t}}^*) \quad (9)$$

Equilibrium price of exports for the non-commodities industry (based on revenue-maximising combination of exports and domestic sales from total supply).

$$P^{X^*}_{NC,t} = {P^E_{NC,t}}^* \left[\frac{1-\phi_{NC}}{\phi_{NC}}\frac{X_{NC,t}}{E_{NC,t}}\right]^{\frac{1}{\sigma^T_{NC}}} \quad (9a)$$

Where: P^W is the world price for that good, ε is the nominal trade-weighted exchange rate and ϕ is the export share.

The dynamics of export markets is quite different for EMMA's commodity industries. As just discussed, for the non-commodities industry, export prices are sticky, so the quantity of exports is demand determined in the short run while the price is supply determined. Essentially, the reverse is true in the two commodities industries where prices are flexible and demand driven. Therefore, the equilibrium relationship is different to equation (9a) for the commodity industries. While this leads to different dynamics for the two types of export markets, the two approaches are equivalent once an equilibrium is reached in which markets clear.

Inventory investment

In modelling inventory investment, EMMA assumes that there is an equilibrium stocks to GDP ratio, as seen in equation (10a). Actual stocks, KST, then adjust to equilibrium stocks, KST^*, in the ECM of equation (10). Inventory investment, $\Delta(KST_t)$, is then calculated as the change in stocks.

Equation 10: Inventory investment

$$\Delta\log(KST_t) = \beta_3[\log(KST_{t-1}) - \log(KST^*_{t-1})] + \beta_4\Delta log(KST_{t-1}) + (1-\beta_4)\Delta\log(Y^*_t) \quad (10)$$

Equilibrium stock of inventories

$$\log(KST^*_t) = \beta_1 + \beta_2 trend_t + \log(Y_t) \quad (10a)$$

Where: KST is the stock of inventories and $trend_t$ is a linear time trend.

5. Approach to Foreign Sector

In modelling international trade, the foreign sector demands exports and supplies imports. Trade in EMMA is modelled under the assumption that Australia approximates a small open economy (SOE). For imports, EMMA adopts the SOE assumption that Australia is a price taker on world markets. However, this is adopted as an equilibrium assumption only. In equilibrium, changes in foreign prices and the exchange rate pass through fully into import prices, P^{M*}, in equation (11a) for non-commodities. The speed of this pass through is determined by an ECM in equation (11).

Equation 11: Price of imports of industry i

$$\Delta\log\,(P^M_{i,t}) = \beta_1\left[\log\left(P^M_{i,t-1}\right) - \beta_2 - \beta_3 trend_{t-1} - \log\left(P^M_{i,t-1}{}^*\right)\right] + \beta_4\Delta\log\,(\varepsilon_t) + \beta_5\Delta\log(P^F_{i,t}) + (1-\beta_5)\Delta\log(P^F_{i,t-1}) \tag{11}$$

Equilibrium price of imports that compete with industry i

$$P^{M*}_{i,t} = \frac{P^F_{i,t}}{\varepsilon_t} \tag{11a}$$

Where: $trend_t$ is a linear time trend, ε is the nominal trade-weighted exchange rate and $P^F_{i,t}$ is trade-weighted foreign prices for industry i.

For exports, the SOE assumption is relaxed somewhat on the basis that some Australian exports are differentiated from the competing exports of other countries, giving some degree of pricing power. This is especially the case for non-commodity exports. This is a diverse category that includes tourism and education services, where Australian product differentiation is important, as well as manufactures where product differentiation is less marked.

In modelling equilibrium exports of non-commodities in equation (12a), the freely estimated price elasticity of demand (ϵ) was negative but inelastic, implying an implausibly high degree of pricing power. This elasticity has been constrained to reflect a more moderate amount of pricing power. Actual non-commodity exports adjust to equilibrium in the ECM of equation (12).

Equation 12: Export demand for non-commodities industry

$$\Delta\log\left(X_{NC,t}\right) = \beta_1\left[\log\left(X_{NC,t-1}\right) - \delta_t - \log(X^*_{nc,t-1})\right] + \beta_3\Delta\left[\log(P^X_{NC,t}/P^F_{NC,t}\,)\cdot\varepsilon_t\right] + \beta_4\Delta\log(Y^F_t) + (1-\beta_4)\Delta\log(Y^*_t) \tag{12}$$

Equilibrium exports for the non-commodities industry

$$X^*_{i,t} = Y^F_t\left(\frac{P^{X\,*}_{i,t}\varepsilon_t}{P^F_{i,t}}\right)^{\epsilon} \tag{12a}$$

Where: δ_t is a time-varying intercept estimated from a state-space model.

It is assumed that Australia is closer to being a price taker for exports from agriculture and mining. This is on the basis that product differentiation is less marked than for non-commodity exports.

6. Approach to government

The approach to government includes the modelling of the government budget and monetary policy.

Government budget

In modelling the government budget, the government is defined as the general government sector. This excludes public corporations, which are treated as part of the business sector. All levels of government – commonwealth, state and local – are consolidated together in a single government sector, although they may be separated as part of future model development work.

The baseline scenario for government expenditures and revenues is taken from official government projections. However, to support alternative scenarios, the government budget is modelled. This budget modelling allows the macroeconomic effects of alternative fiscal settings to be explored, including settings that change tax rates or expenditures. It also means that other model simulations take into account the likely broad effects on the budget when there are fluctuations in prices, incomes and expenditures.

For the government budget modelling, government expenditures on goods and services are assumed to vary with GDP in the long run. More specifically, equilibrium real government consumption expenditure, G^{c^*}, (exclusive of depreciation) is specified as a fixed share of real GDP in equation (13a). Actual government consumption, G^c, then adjusts to this equilibrium in the ECM of equation (13). Real government investment expenditure, G^i, is modelled in an analogous way in equations (14a) and (14).

Equation 13: General government consumption (exclusive of depreciation)

$$\Delta\log(G_t^c) = \beta_2\left(\log(G_{t-1}^c) - \log(G_{t-1}^{c^*})\right) + \Delta\log(Y_t^*) \qquad (13)$$

Equilibrium government consumption (exclusive of depreciation)

$$\log(G_t^{c^*}) \equiv \beta_1 + \log(Y_t) \qquad (13a)$$

Equation 14: General government investment

$$\Delta\log(G_t^i) = \beta_2\left(\log(G_{t-1}^i) - \log(G_{t-1}^{i^*})\right) + \Delta\log(Y_t^*) \qquad (14)$$

Equilibrium government investment (exclusive of depreciation)

$$\log(G_t^{i^*}) \equiv \beta_1 + \log(Y_t) \qquad (14a)$$

This approach to government expenditure is adopted so that the model can converge to a balanced growth path in the long run, with GDP and its components, including government final demand, growing at the same rates. The equilibrium government share of GDP is adjusted as a model input.

Tax revenues are divided into five categories including personal income tax, company income tax, product taxes net of subsidies, other production taxes net of subsidies and taxes on non-residents. These taxes are modelled in identities that apply an effective tax rate to a model construct for the tax base. Hence, at the margin, tax revenues vary proportionately with each tax base. Future work may consider incorporating progressivity of the tax system into the model's shock dynamics. Progressivity in the income tax system is captured in the model's baseline projections by using the Government's official projections for revenue. Changes in tax rates flow through to changes in prices and incomes.

While this modelling of the government budget suffices for shorter-term analysis, for longer-term scenario analysis it is augmented to ensure that the government budget is sustainable. This involves specifying a rule in which fiscal policy adjusts gradually to stabilise government debt relative to GDP in the long term. Debt stabilisation relative to GDP will only be achieved without such fiscal adjustments under the condition that the nominal interest rate on government debt is lower than the growth rate in nominal GDP. While that condition holds in the current low interest rate environment, this need not always be true. Hence, EMMA includes a fiscal policy rule to ensure that the government budget is sustainable, as is standard practice in macroeconomic models.

The EMMA fiscal policy rule defines a trend, B^*, for the ratio of government net debt, B, to trend nominal GDP, YZ^* (which is calculated as potential GDP Y^* inflated by prices growing at the mid-point of the RBA's target band). While in practice the trend may be restored through a variety of measures such as tax increases or expenditure cuts, for scenario analysis EMMA makes the common simplifying assumption that fiscal sustainability is achieved through gradual adjustments in the average rate of personal income tax, $\Delta\tau^{HH}$. This considerable simplification of choosing a single swing fiscal instrument is common in macroeconomic models and means fiscal policy rules are often calibrated, as is the case here, rather than estimated. Other swing instruments can also be used, depending on the simulation.

In any case, whatever the choice of swing instrument, the model can be used to simulate the use of virtually any type of fiscal measures to achieve debt stabilisation. To the extent that other fiscal measures are used to achieve debt stabilisation, little or no adjustment may be required in whatever is the designated swing instrument.

In general B^* is set to be consistent with the trajectory for net debt in the baseline projections based on current government policy. This trend share can be readily adjusted as a model input. Given the use of the fiscal policy rule for scenario and policy analysis, B^* can be used to ensure that net debt as a share of GDP returns to its projected level under the baseline projections.

In the fiscal policy rule (equation 15), the rate of personal income tax increases if debt is above B^*or if this debt gap increases. The change in the debt gap is included in addition to the level of the debt gap because this improves the performance of adjustments in the tax rate in achievingB^*. The values of the two parameters appearing in the rule are chosen based on model simulation properties. Specifically, in model simulations that open a debt gap, the fiscal policy rule closes that gap in about 10 years under the chosen parameter values. The two parameter values can be adjusted to achieve B^*either more slowly or more quickly. The values of the two parameters are the same as those that were used by the TRYM model.

Equation 15: Fiscal policy rule

$$\Delta\tau_t^{HH} = -0.007\left(\frac{B_t}{YZ_t^{\ *}} - B_t^*\right) - 0.12\,\Delta\left(\frac{B_t}{YZ_t^{\ *}} - B_t^*\right) \tag{15}$$

The fiscal policy rule is designed to achieve the stated purpose of ensuring fiscal sustainability in the long run during model simulations. In the short run, counter-cyclical fiscal policy can be modelled in a flexible way by varying a wide range of individual fiscal inputs. In short, the approach to modelling fiscal policy is flexible rather than prescriptive.

Monetary policy

Besides a fiscal policy rule to achieve long-run stabilisation of government debt, EMMA also requires a monetary policy rule to ensure long-run stabilisation of inflation. Macroeconomic models often use the Taylor rule for that purpose, which is shown below using textbook notation.

$$i_t = \pi_t + r_t^* + \alpha_\pi\,(\pi_t - \pi_t^*) + \alpha_y\,(\log(Y_t) - \log(Y_t^*))$$

Under the Taylor rule, monetary policy is used to achieve a chosen inflation target. Monetary policy is said to be tight or loose when the policy interest rate, i, is above or below a neutral nominal rate. The neutral nominal rate is calculated as a neutral real rate,[11] r^*, plus the inflation rate, π. Monetary policy is tighter when inflation, π, is above its target, π^*, or output, y, exceeds its potential, y^*. That is, monetary policy is driven by inflation and output gaps.

EMMA uses the version of the Taylor rule that appears in the RBA's MARTIN model (Ballantyne et al., 2020). MARTIN varies the standard Taylor rule shown above by replacing the output gap with an unemployment gap, calculated as the difference between the unemployment rate and the NAIRU. It also assumes that the actual interest rate adjusts gradually, rather than contemporaneously, to the rate indicated by the basic rule. The adjustment is also influenced by the change in the unemployment gap (the unemployment rate U relative to the NAIRU U^*).

$$i_t = \beta\, i_{t-1} + (1-\beta)\,[\pi_t + r_t^* + \alpha_\pi\,(\pi_t - \pi_t^*) + \alpha_u\,(U_t - U_t^*)] - \alpha_c \Delta_2 U_t$$

This rule as it appears in EMMA can be seen in equations (16a) and (16). The interest rate from the rule is shown in equation (16a), while the adjustment of the actual interest rate to that rule is shown in equation (16).

Equation 16: Monetary policy rule for cash rate

$$\Delta i_t = 0.3\,(i_t^* - i_{t-1}) - \Delta_2(U_t - U_t^*) \tag{16}$$

$$i_t^* = r_t^* + \pi_t^{TM} + (\pi_t^{TM} - \pi_t^*) - 2\,(U_t - U_t^*) \tag{16a}$$

11 As discussed in the section on Financial Markets, EMMA assumes uncovered interest parity (UIP). The combination of UIP and relative PPP means that the neutral real rate appearing in the monetary policy rule should ultimately be driven by the foreign real interest rate. This is taken into account in model simulations by adjusting the neutral real rate in line with any shock to the foreign real interest rate.

All of the parameter values in the policy interest rate rule are imposed rather than estimated. Ballantyne et al. (2020) state that this approach is taken 'in light of the well-known difficulties in estimating the parameters of monetary policy reaction functions'.

The MARTIN version of the Taylor rule is one way of representing monetary policy in EMMA. The alternative way is to use optimal control of monetary policy. Under optimal control, there is still the same general idea that monetary policy is driven by the inflation and unemployment gaps. There is also the same aim of adjusting the policy interest rate gradually rather than abruptly.

To achieve the optimal trade-off between these three potentially conflicting targets, optimal control minimises a loss function. That function includes the squared inflation gap, the squared unemployment gap and the squared change in the policy interest rate. Subjective weights are attached in combining these three sources of loss into a single measure of loss. A discount rate is used so lower weights are placed on losses the further that they occur into the future. In the literature, there are two versions of optimal control, closed loop and open loop.

Under closed loop optimal control, the Taylor rule can continue to be used. Multiple simulations are conducted in which the model is subjected to a typical range of shocks. The loss function is used to determine the optimal values for the Taylor rule parameters (β, α_π and α_u). This closed loop approach has the advantage that the policy approach can be easily understood from the Taylor rule and the disadvantage that this may be overly restrictive.

Instead, EMMA uses open loop optimal control. This discards the Taylor rule. Instead, the loss function is used in determining the optimal entire path for the policy interest rate, and this is done separately for each shock. To avoid the problem of time inconsistency that may occur using closed loop optimal control, for any given shock, the monetary authority is assumed to commit to a path for the policy interest rate, and not re-optimise along that path.

7. Approach to markets

The main markets in EMMA include the product markets for the three industries, the labour market and financial markets. Prices are sticky in some of these markets and flexible in others, but in the long run all markets clear. This section describes the process of price adjustment to clear each market, taking the product, labour and financial markets in turn.

Product markets

As discussed previously, in modelling price adjustment in product markets, EMMA makes a distinction between its non-commodity industry and its two commodity industries, agriculture and mining. In broad terms, prices are sticky for the non-commodity industry, so in a Keynesian short run the representative business temporarily operates off its equilibrium supply curve by varying its factor utilisation, creating cyclical variation in productivity and total hours work, in order to accommodate demand at the prevailing price level. However, in the two commodity industries, prices are flexible, so that the representative business can vary its short-run supply.

In each industry a distinction can be made between the product market for local sales, E, and the product market for exports, X, as seen in Figure 1. These two end uses of domestic production, Y, are linked using CET technology. In equilibrium, the representative business chooses the combination of local sales and exports that maximises revenue given their relative prices.

The operations of the market for non-commodities were partly explained in the section on the approach to businesses. It was pointed out that the stickiness in prices refers to local sales. More specifically, the actual price of local sales, P^E, adjusts gradually to the marginal cost of its supply, P^{E^*}. At that price, the market for local sales clears, with profit maximising supply matching price-sensitive demand.

The price of exports of non-commodities is similarly sticky. In this case, the actual price of exports, P^X, adjusts gradually to the equilibrium price, P^{X^*}, at which the revenue maximising combination of exports and local sales is produced. As the actual price adjusts, export demand responds reflecting the price elasticity referred to in the discussion of the foreign sector.

In contrast, price adjustment is flexible and export-driven in the two commodity industries. Australia is considered to be almost a price taker on these export markets. To reflect this idea, inverse export demand equations are specified in which the Australian share of the world market has a very limited effect on price. For mining, changes in world price (or the exchange rate) flow through contemporaneously into Australian export prices. For agriculture, the flow through is almost as quick, being spread over two quarters. Thus, prices are highly flexible in the two commodity export markets.

While in the case of the non-commodity industry the principle of revenue maximisation over the two outputs was used in modelling the flow through of the price of local sales into export prices, this approach is reversed for the two commodities industries. That is, the actual price of local sales adjusts gradually to the shadow price at which the revenue maximising combination of exports and local sales are produced. This means that in the two commodities industries, price adjustment is less rapid for local sales than for exports.

Labour market

Besides the product market for non-commodities, prices are also sticky in the labour market. In an inflation expectations-augmented Phillips Curve, quarterly wages growth is modelled as the sum of equilibrium and disequilibrium components in equation (17).

Equation 17: Wage equation

Signal equation:

$$\begin{aligned}\Delta \log(W_t) - \Delta \log(Z_t) - \pi_t^{TM} &= \beta_1(\pi_t^e - \pi_{t-1}^{TM}) + D_{<1993}\beta_2\left(\frac{U_t - U_t^*}{U_t}\right) + D_{\geq 1993}\beta_3\left(\frac{U_t - U_t^*}{U_t}\right) \\ &+ \beta_4\left(\frac{\Delta U_{t-1}}{U_t}\right) + \beta_5(\Delta \log(Z_t^*) - \Delta \log(Z_t)) + \upsilon_t\end{aligned} \tag{17}$$

State equation:

$$U_t^* = U_{t-1}^* + \xi_t$$

Household expected inflation (percentage per year)

$$\Delta\pi_t^e = \beta_1(\pi_{t-1}^e - \pi_t^*) + \beta_2\,(\pi_{t-1}^{TM} - \pi_{t-1}^e) \tag{17a}$$

Where: $\Delta\log(W)$ is hourly average earnings growth, $\Delta\log(Z)$ is growth in productivity, $\Delta\log(Z^*)$ is growth in trend productivity, π_t^e is inflation expectations, π^{TM} is trimmed mean inflation, D is a structural break (prior to or since the March quarter 1993), U_t is the unemployment rate, U_t^* is the NAIRU, with $\upsilon_t \sim (0, \sigma_\upsilon^2)$ and $\xi_t \sim (0, \sigma_\xi^2)$, and π^* is the inflation target.

Equilibrium wages growth is equal to the sum of expected price inflation and expected productivity growth. These expectations are modelled as a weighted combination of recent observations and long-term factors. In effect, expected price inflation depends on price inflation in the previous quarter[12] and the mid-point of the RBA's inflation band. As such, this approach is partly backward looking, but also assumes households have some understanding that monetary policy targets inflation. However, households are not assumed to have the complete model understanding that would be implied under the alternative assumption of rational expectations. Expected productivity growth depends on both current and trend productivity growth.

Disequilibrium wages growth depends on the gap between the unemployment rate and the NAIRU. When the unemployment rate is below the NAIRU, the tight labour market adds to wages growth. This pushes down on labour demand until the unemployment gap is removed, with the unemployment rate equal to the NAIRU. Wages are sticky rather than flexible, so this adjustment process may take several years. The wage equation also contains a hysteresis effect, with wages growth being stimulated not only by a low unemployment rate, but also by a falling unemployment rate.

Further details on the wage equation used in EMMA can be found in Ruberl et al. (2021).

---

12 In EMMA, price inflation in the previous quarter adds to wages growth through two channels. First, there is a direct effect in the wage equation, equation 17. Second, there is an indirect effect operating via household inflation expectations in equations 17a. The text simplifies by describing these two effects as a single effect.

Financial markets

Similar to commodity export markets, prices are fully flexible in financial markets. Further, financial market expectations are assumed to be model consistent. This combination of assumptions provides an operationally simple way of capturing the real-world phenomenon that asset prices jump when there is new information. The assumption of price flexibility allows asset prices to jump and the assumption of model consistent expectations allows all new information to the model to be taken into account in determining the size of the jumps.

The assumption of model consistent expectations is viewed as an approximation to reality as it is not suggested that financial markets literally use EMMA to form their expectations for the future. Model consistent expectations are used below in both the uncovered interest parity condition and the term structure equation.

In modelling the choices made between financial assets, it is assumed that the assets are perfect substitutes, after allowing for risk premiums. This applies to short-term and long-term domestic debt and short-term foreign debt. These leads to two equations that equate risk-adjusted ex ante rates of return.

The one quarter ex ante returns for domestic and foreign short-term debt securities are equated in an uncovered interest parity condition. This sets the return from investing one dollar in a domestic short-term security (at the interest rate i) equal to the return from investing in an equivalent foreign security (at the interest rate i^F) after adjusting for the expected movement in the exchange rate, ε.

$$1+\frac{i_t}{400}=\left(1+\frac{i_t^F}{400}\right)\frac{\varepsilon_t}{\varepsilon_{t+1,t}}$$

The overnight cash rate is used for the return on the domestic security. Because EMMA is a quarterly model, in strict terms one should use a security with a term of three months. However, the cash rate is already included in the model as the instrument of monetary policy and it was decided not to add complexity by including two different short-term interest rates in the model.

As the cash rate is already determined in the monetary policy rule, the uncovered interest parity condition is used as the equation for the exchange rate. To give effect to this, the above equation is logged and then inverted to solve for the exchange rate. A risk premium, $risk\_\varepsilon$, is then included to modify the return on the foreign security and obtain equation (18) of the model.

Equation 18: Exchange rate

$$log(\varepsilon_t)=log(\varepsilon_{t+1,t})+\ log\left(1+\frac{i_t}{400}\right)-log\left(1+\frac{i_t^F}{400}\right)-risk\_\varepsilon_t \tag{18}$$

The return from holding a long-term security is equated with the expected return from holding a sequence of short-term securities over the same term in a term structure equation. In the case of EMMA, the 10-year bond rate, i^{10}, is used as the return on the long-term security, while i is used as the return on the short-term security and again is treated as if it were a 3-month interest rate.

It is cumbersome to implement the term structure equation in raw form because it involves 40 expectations terms, made up of the sequence of expected returns on the 3-month security over a 10-year period. EMMA uses a modified form of the term structure, as described in Powell and Murphy (1995). The modified form replaced uniform weights on expected future interest rates with geometrically declining weights. After manipulation, this led to the following term structure equation in which there is a single expectation for the long-term interest rate in place of multiple expectations for the short-term interest rate.

$$i_t^{10} = (1 - 0.95)\, i_t + 0.95\, i_{t+1,t}^{10}$$

It can be argued that, for the purpose of macroeconomic models, it is more appropriate to use this modified form, with weights on expected future short-term interest rates that decline into the future, than the original raw form. This is because the long-term bond rate is typically introduced into macro models to help drive investment decisions. From microeconomic foundations, investment decisions depend on expected future short-term interest rates with weights that decline into the future because of time discounting and depreciation.

EMMA uses the modified expectations theory of the term structure above and allows for a risk premium, $risk\_i10$. This gives equation (19).

Equation 19: 10-year bond rate

$$i_t^{10} = risk\_i10_t + (1 - 0.95)i_t + 0.95\left(i_{t+1,t}^{10} - risk\_i10_{t+1,t}\right) \quad (19)$$

The investment equations in EMMA use the real 10-year bond rate. Thus, EMMA requires an expected 10-year inflation rate to convert the nominal 10-year bond rate to the real 10-year bond rate. In equation 19, this 10-year expected inflation rate, $\pi^{e,10}$, is modelled to depend on the current inflation rate in the same way that the 10-year bond rate was modelled to depend on the short-term interest rate. This follows Powell and Murphy (1995).

Equation 20: 10-year expected inflation

$$\pi_t^{e,10} = (1 - 0.95)\pi_t + 0.95\, \pi_{t+1,t}^{e,10} \quad (20)$$

Flow of funds

EMMA models financial flows in a more detailed way than might appear from the above or from the 20 key equations listed. For this purpose, EMMA uses the ABS Australian National Accounts financial and wealth data, which are published each quarter separately from the main quarterly national accounts release.

There are four agents within the framework: a household, firm, the government and the rest of the world. Agents hold two types of assets and liabilities: debt and equity. The framework captures party counter-party assets and liabilities held between each of the agents in the two asset classes. Property income flows are paid between agents reflecting their stock of debt and equity assets and liabilities. Reflecting the party counter-party nature of these transactions, property income flows net to zero when summed across the four agents.

Domestic agents fund any shortfall between their saving and investment by borrowing from other agents. In the same way, any excess of saving over investment is loaned to other agents. Every asset acquired by one agent represents another agent's liability. The sum of the net lending positions of all of the agents in the model is zero in all periods.

The accounting system that supports the flow of funds in the model is large. Most of these equations are identities.

8. Steady state

This section discusses EMMA's long-run or steady state properties. It begins by considering under what conditions EMMA converges to a balanced growth path. It then considers the causal structure of EMMA in its long-run equilibrium, which is useful in understanding the model's long-run simulation properties.

Steady state growth

EMMA is designed so that, in the long run, it is possible for it to converge to a path of steady, balanced growth. That path resembles that of the Solow-Swan growth model in that output growth, $\Delta \log(Y_t)$, is determined by growth in the effective supply of labour. This is made up of growth in the labour supply, n,[13] plus growth in labour efficiency, λ. When growth is balanced, this same rate of output growth is observed in all of EMMA's industries.

Whether EMMA actually converges to such a path of steady, balanced growth in long-run projections depends on the settings for some key model inputs. Balanced growth is a useful starting point for analysis. EMMA can further be used as a tool in developing a clear economic understanding of the implications of any likely departures from steady, balanced growth in the long run. Such an understanding is important for the preparation of the Intergenerational Report and other long-term economic analysis.

Under balanced growth, all real variables grow at the same rate in the steady state. To make this possible, EMMA's equations are linear in real variables. For example, the industry production functions make the common assumption of constant returns to scale. Similarly, the consumer demand system used for allocating consumption expenditure across consumption categories assumes that all four income elasticities of demand are unity and consumption grows in line with output in the long run. This assumption seems reasonable for the broad consumption categories used in EMMA, although in a more disaggregated model it may be desirable to distinguish between necessities and luxuries.

EMMA's use of an A-M consumption function is also compatible with balanced growth. Under the A-M consumption function, the long-run growth rate in consumption matches the long-run growth rate in income or output. If instead the Euler equation was used to model consumption, as in DSGE models, this matching would generally not occur under the EMMA assumption of perfect international capital mobility. A knife-edge condition on the rate of time preference would be needed to force matching, as explained in the section on the approach to households. For further explanation of the long-run properties of the two consumption functions, see the section on the approach to households.

As noted above, whether EMMA actually converges to a balanced growth path in long-run projections depends on the settings for certain model inputs. The two main areas to consider are the inputs for the non-produced factors of production (labour and fixed factors) and the international economy. The balanced growth requirements for these two areas of inputs are now discussed in turn. The reasonableness of those requirements is then evaluated. Balanced growth requires that labour efficiency grows at a constant rate. Further, that rate needs to be uniform

13 $n = \Delta \log\left(POP_t^*\right) + \Delta \log\left(H_t^*\right) + \Delta \log\left(\rho_t^*\right)$

across industries. In EMMA, this means that the trend in labour efficiency within each industry grows at the same rate.

Balanced growth also requires that employment grows at a constant rate. In EMMA, the main requirement for this is that the working-age population grows at a constant rate, n.

Besides labour, the other type of non-produced factor of production in EMMA is the fixed factor. This fixed factor is industry-specific, and the industries with fixed factors are agriculture, mining and ownership of dwellings. For growth to be balanced, in each of these three industries the effective supply of the fixed factor would need to grow at the same rate as the effective supply of labour.

EMMA is set up so that this balanced growth condition holds by default in the ownership of dwellings industry. The supply of the fixed factor, housing land, F_{ODW}, grows with the working age population, and the efficiency of housing land, λ_{ODW}, grows with labour efficiency. This is analogous to the way labour inputs are projected to grow in other industries.

Growth in the effective supplies of the other two fixed factors, agricultural land and mineral resources, is projected in a different way in EMMA. The default assumption is that the actual supplies of these two fixed factors do not grow with population, but rather are unchanged. Balanced growth is still possible, but requires that the growth rates for the efficiency of these two factors, λ_{AG}, and λ_{MIN}, are set equal to the growth rate in the effective supply of labour supply, $n + \lambda$. If instead the efficiency of these two fixed factors only grows at the same rate as labour efficiency, a Malthusian element would be introduced into long-term projections. The significance of this brake increases or decreases into the future if the elasticity of substitution between the fixed factor and labour is below or above unity. In EMMA, this elasticity is below unity. As noted above, while these settings for growth in the fixed factor are required to achieve balanced growth, these conditions need not be met. Rather EMMA is a tool in developing a clear economic understanding of the implications of any likely departures from steady, balanced growth in the long run.

Settings for the international economy also need to be compatible if balanced growth is to occur. The four foreign prices appearing in EMMA[14] would all need to inflate at the same rate. Otherwise, trends in foreign relative prices would lead to changing patterns of trade, resulting in different industries growing at different rates.

Finally, EMMA uses a major trading partner (MTP) growth index, Y^F, to scale export demand for each industry. For balanced growth to occur, this foreign activity index would need to grow at the same rate as local output, $n + \lambda$. If foreign growth were faster or slower than domestic growth, the domestic economy would steadily become more or less open, leading different industries to grow at different rates.

Historically, growth in labour efficiency has differed between industries, contributing to imbalances in growth. However, if this divergence were to persist indefinitely, the ultimate consequences can be implausible, especially in an open economy. This suggests that it may be reasonable to assume that the historic differences between industries in their rates of growth in labour efficiency gradually erode away over a decade or more when projecting into the future. This means that balanced growth may still be achieved, but in the more distant future.

---

14 The four foreign prices are used as follows: P^F_{ag} in modelling demand for agriculture exports, P^F_{min} in modelling demand for mining exports and supply of mining imports, P^F_{nc} in modelling demand for non-commodity exports and P^F in modelling supply of agriculture imports and non-commodity imports.

This focus in the design of EMMA on its steady state properties is useful in avoiding some modelling pitfalls. These pitfalls include projecting departures from steady, balanced growth in the long run because of accidents in model design, such as unintended departures from linear homogeneity in real variables in some equations, or a lack of consistency in the approach to setting model inputs. With those pitfalls avoided, EMMA can be used as a tool in developing a clear economic understanding of any likely departures from steady, balanced growth in the long run.

Steady state structure

In long-run equilibrium, EMMA has a structure that is almost recursive, which assists in understanding its long-run simulation properties. This almost recursive structure begins with the production technology, then flows to household consumption and the associated budget constraint and finally to trade volumes and the real exchange rate. Those three stages are now discussed in turn.

The production and trade technology of Figure 1 is the first stage of the recursive structure. While there is a separate production technology for each of EMMA's three industries, for simplicity this discussion of the model's long-run structure abstracts from that industry detail.

In EMMA, the effective supply of labour is determined by the so-called "Three Ps" of population, participation and productivity. In the long run, each of the 3Ps can be regarded as inputs to the model. The effective supply of labour in turn drives the effective use of labour because the unemployment rate is driven to the NAIRU by the wage equation. Thus, the explanation of long-run equilibrium in EMMA can begin with this effective labour input, NH.

Under the modelling assumptions, in the long run, the scale of production is essentially determined by effective labour input. Thus, in EMMA, the 3Ps broadly drive the scale of the economy. That is, when effective labour input expands, the other main variables in Figure 1 (capital (*K*), domestic output(*D*), imports (*M*), exports (*X*), consumption(*C*), investments (*I*) and government final demand (*G*)) will eventually expand by the same proportion. The two main qualifications to this also arose in the above discussion on the conditions for balanced growth in the steady state.

First, as shown in Figure 1, there are fixed factors of production in the agriculture and mining industries. If these fixed factors are assumed to expand in tandem with the rise in effective labour input, then the result that the 3Ps drive the scale of the economy will continue to hold. Otherwise, the fixed factors will act as a mild brake on the general expansion in the economy when effective labour supply expands.

Second, when effective labour supply expands, the induced general expansion in the economy includes the trade volumes, exports and imports. This can occur in a frictionless way if Australia is assumed to be a SOE with an exogenous terms-of-trade. However, as noted previously, in EMMA Australia only approximates an SOE. In particular, for the rest of the world to absorb an expansion in Australian exports, some endogenous fall in the terms of trade is required, particularly for non-commodities. This change in relative prices will have some effects on the pattern of economic activity in EMMA.

For simplicity, the remaining discussion here of the model's steady state properties makes the simplifying assumption that the scale of production in EMMA is driven by the level of effective labour input.

The starting point in understanding the long-run properties of EMMA is the implied long-run zero pure profit (ZPP) condition involving trade values. Specifically, in Figure 1, the value of total supply consisting of domestic output and imports will equal the value of total use made up of exports, and expenditure, E. For present purposes, one can also make the simplifying assumption that the value of exports equals the value of imports to achieve external balance, although this will only by exactly true in an equilibrium in which net foreign liabilities are zero. In any case, under these assumptions the terms of trade, P^X/P^M, entirely determines the price of output relative to the price of expenditure, P^D/P^E. Hence, a higher terms of trade raises the price of output relative to the price of expenditure.

As discussed previously, EMMA's monetary policy rule targets consumer price inflation. For present purposes, we can think of that as determining the price of expenditure. Thus, when an increase in the terms of trade increases P^D/P^E, this is likely to occur mainly via an increase in P^D than via a decrease in P^E.

Next, there is also a ZPP condition for factor use. As discussed earlier, this condition follows from the use of Tobin's-q theory of investment under which the actual rate of return on capital equals the required rate of return in the long run. In terms of Figure 1, in the long run, the value of domestic output will equal the cost of capital plus the cost of labour. However, the price of domestic output and capital have already been determined by P^D and P^E respectively. Hence, the role of the ZPP condition for factor use is to determine the wage, W. That is, in EMMA, the real wage will adjust in the long run to ensure the actual rate of return on capital matches the rate required on world capital markets.

For example, if labour productivity rises, the real wage will eventually rise proportionately, restoring the ZPP condition for factor use. Alternatively, if the terms of trade rises, increasing the price of output, P^Y, relative to the price of new investment the potential pure profits will be neutralised by a rise in the wage relative to the price of output.

Next, the marginal product of labour condition equates the marginal product of labour, which depends on the capital-to-labour ratio, with the real wage. As explained previously, this condition applies once the price of domestic output has adjusted to equal the marginal cost of production. With employment and the real wage already determined above, the capital stock adjusts to maintain the marginal product of labour condition. For example, if labour input rises with population, the capital stock will rise proportionately, maintaining the ratio of the capital stock to effective labour input.

Next, the main production function determines domestic output. From Figure 1, domestic output depends on the inputs of capital and effective labour which have already been determined.

Then, the process of capital accumulation determines the level of investment. With the capital stock already determined, investment needs to cover depreciation plus growth in the capital stock in line with growth in the real economy. This completes the first stage of the recursive model structure in long-run equilibrium.

In the second stage of the recursive structure, household consumption and wealth are determined as explained in the section on the approach to households. There it was shown that the A-M consumption function combined with the long-run household budget constraint simultaneously determine household consumption and household wealth as ratios to after-tax labour incomes, in the long run. Labour incomes have already been determined in the first stage through wages and effective labour.

Government final demand can also be determined as it is modelled as a ratio to domestic output which was determined in the first stage. From Figure 1, consumption, investments and government final demand can now be summed to determine total real expenditure, E.

The third stage deals with trade volumes and the real exchange rate. Figure 1 shows are exports and imports enter the production technology.

In EMMA, the profit-maximising producer chooses the ratio of imports to domestic output that minimises the cost of total supply. This means that imports depend on domestic output and price of domestic output which are already known from the first stage, as well as the foreign currency price of imports, which is exogenous. However, the choice of imports also depends on the exchange rate which is not known yet.

Similarly, the profit-maximising producer chooses the ratio of exports to home market supply that maximises revenue from total supply. This means that exports depend on total expenditure and the price of expenditure, which are already known from the second and first stages respectively, as well as the foreign currency price of exports, which is exogenous. However, the choice of exports also depends on the exchange rate, which is not known yet.

The final model relationship in the third stage is the trade technology of Figure 1. This links the supply of domestic output and imports to their use for home expenditure and exports. Domestic output was determined in the first stage and domestic expenditure in the second stage. Thus, the trade technology links the volume of exports and imports. Indirectly this reflects a requirement for external balance that has been introduced via the determination of domestic expenditure in the second stage.[15]

Thus, in the third stage there are three relationships – for import demand, export supply and the trade technology. These three relationships simultaneously determine exports, imports and domestic expenditure. In effect, the exchange rate adjusts until the level of exports relative to the level of imports is consistent with external balance.

15 In the second stage, E depends on C which depends on the household budget constraint which reflects a requirement for external balance.

9. Conclusion

A macroeconomic model is never finished. This paper has outlined the structure of Version 1.0 of EMMA. Several areas for further development have been identified in this paper. The model will continue to be developed over time to ensure it remains fit-for-purpose to meet the needs for the model to analyse the evolving economic and policy environment. This paper outlines the first version of the model. Further details on the model's equations, empirical properties and dynamics will be published over time.

REFERENCES

Aron, J, Duca, J, Muellbauer, J, Murata, K, and Murphy A 2012, 'Credit, Housing Collateral, and Consumption: Evidence from Japan, the U.K., and the U.S.', *Review of Income and Wealth*, 58:3, pp. 397–423.

Auerbach AJ, Kotlikoff LJ 1987. Dynamic Fiscal Policy. Cambridge University Press, Cambridge, U.K.

Ballantyne, A, Cusbert, T, Evans, R, Guttmann, R, Hambur, J, Hamilton, A, Kendall, E, McCririck, R, Nodari, G, and Rees, D 2020, 'MARTIN Has Its Place: A Macroeconometric Model of the Australian Economy', *Economic Record*, 96, pp. 225–251.

Blanchard, O 1985, 'Debt, Deficits, and Finite Horizons', *Journal of Political Economy*, 93:2, pp. 223–247.

Blanchard, O 2018, 'On the future of macroeconomic models', *Oxford Review of Economic Policy*, 34, pp. 43–54.

Christiano, L, Eichenbaum, M, and Evans, C 2005, 'Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy', *Journal of Political Economy*, 113:1, pp. 1-45.

Coenen, G, Erceg, C, Freedman, C, Furceri, D, Kumhof, M, Lalonde, R, Laxton, D, Lindé, J, Mourougane, A, Muir, D, Mursula, S, Resende, C, Roberts, J, Roeger, W, Snudden, S, Trabandt, M, and Veld, J 2012, 'Effects of Fiscal Stimulus in Structural Models', *American Economic Journal: Macroeconomics*, 4, pp. 22–68.

Downes, P., Hanslow, K. and Tulip, P. (2014), *The Effect of the Mining Boom on the Australian Economy'*, RBA Research Discussion Paper 2014-08, Reserve Bank of Australia, Sydney.

Dungey, M, and Pagan, A 2000, 'A Structural VAR Model of the Australian Economy', *Economic Record*, 76, pp. 321–42.

Dungey, M, and Pagan, A 2009, 'Extending a SVAR Model of the Australian Economy', *Economic Record*, 85, pp. 1–20.

Fukač, M, and Pagan, A 2010, 'Limited Information Estimation and Evaluation of DSGE Models', *Journal of Applied Econometrics*, 25, pp. 55-70.

Fukač, M, and Pagan, A 2011, 'Structural Macroeconometric Modelling in a Policy Environment', in Ullah, A. and Giles, D. (eds), *Handbook of Empirical Economics and Finance*, Chapman and Hall/CRC, Boca Raton.

Gibbs, C, Hambur, J, and Nodari, G 2018, 'DSGE Reno: Adding a Housing Block to a Small Open Economy Model', *Research Discussion Paper 2018-04*, Reserve Bank of Australia, Sydney.

Grohé, S, and Uríbe, M 2003, 'Closing small open economy models', *Journal of International Economics*, 61, pp. 163–185.

Groth, C, and Khan, H 2010, 'Investment Adjustment Costs: An Empirical Assessment', *Journal of Money, Credit and Banking*, 42:8, pp. 1469-1494.

Hawkins, J 2005, 'Economic forecasting: history and procedures', *Economic Roundup Autumn 2005*, Australian Treasury: Canberra.

Higgins, C 1970, 'A model for national income forecasting', paper presented to ACT branch of Economic Society of Australia.

Higgins, C and Fitzgerald, V 1973, 'An econometric model of the Australian economy', Journal of Econometrics, 1, pp. 229-65.

Kaldor, N 1963, *Capital Accumulation and Economic Growth*. MacMillan, London.

Kudrna, G, and Woodland, A 2011, 'An inter-temporal general equilibrium analysis of the Australian age pension means test', *Journal of Macroeconomics*, 33, pp. 61–79.

Laforte, J 2018, 'Overview of the Changes to the FRB/US Model', *Fed Notes*, Board of Governors of the Federal Reserve System, Washington, December 7, 2018.

Gustafsson, L 2021, 'Australian Labour Force Participation: Historical Trends and Future Prospects', *Treasury Working Paper*, 2021-2, Australian Treasury, Canberra.

Murphy C 1988, 'An overview of the Murphy Model', in Burns M and Murphy (eds), Macroeconomic Modelling in Australia, Australian Economic Papers, Supp.

McKibbin, W, and Wilcoxen, P 1999, 'The theoretical and empirical structure of the G-Cubed model', *Economic Modelling*, 16, pp. 123–148.

Murphy, C 2017, *Review of economic modelling at the Treasury*, Independent Economics, Canberra.

Murphy, C 2020, 'Decisions in Designing an Australian Macroeconomic Model', *Economic Record*, 96, pp. 252–270.

Murphy, C, Bright, I, Brooker, R, Geeves, W, and Taplin B 1986, 'A Macroeconometric Model of the Australian Economy for Medium-term Policy Analysis', Office of the Economic Planning Advisory Council Technical Paper, No. 2, Canberra.

Pagan, A, and Robinson, T 2016, 'Investigating the Relationship between DSGE and SVAR Models', *NCER Working Paper 112*, National Centre for Econometric Research, Brisbane.

Pagan, A, and Wilcox, D 2016, 'External Review – Reserve Bank of Australia Economic Group Forecasts and Analysis', Reserve Bank of Australia

Pagan, A 2019, 'Australian Macro-Econometric Models and Their Construction - A Short History', *CAMA Working Paper 50/2019,* Centre for Applied Macroeconomic Analysis, Canberra.

Powell, A, and Gruen F 1968, 'The Constant Elasticity of Transformation Production Frontier and Linear Supply System', *International Economic Review*, 9, pp. 315-328.

Powell, A, and Murphy, C 1995, *Inside a Modern Macroeconomic Model – A Guide to the Murphy Model*, 1st edition, Springer-Verlag, Berlin.

Rees, D, Smith, P, and Hall, J 2016, 'A Multi-sector Model of the Australian Economy', *Economic Record*, 92, pp. 374–408.

Ruberl, H, Ball, H, Lucas, L and Williamson, T 2021, 'Estimating the NAIRU in Australia', *Treasury Working Paper,* 2021-01, Australian Treasury, Canberra.

Saxegaard, M 2017, 'The Use of Models in Finance Ministries – An Overview', *Work Note* 2017/1, Norwegian Ministry of Finance.

Schmitt-Rohe, S and Uribe, M 2003, 'Closing small open economy models', *Journal of International Economics*, 61, pp. 163-185.

Sims, C 1980, 'Macroeconomics and Reality', *Econometrica*, 48, pp. 1–48.

Simes, R and Horn, P 1988, 'Design of the NIF88 model', *Australian Economic Papers*, supplement to volume 27, June, pp. 155-70.

Smets, F, and Wouters, R 2007, 'Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach', *American Economic Review*, 97, pp. 586–606.

Stock, J, and Watson, M 2001, 'Vector Autoregressions', *Journal of Economic Perspectives*, 15, pp. 101–15.

Taplin, B, Jilek, P, Antioch, L, Johnson, A, Parameswaran, P, and Louis, C 1993, *Documentation of the Treasury Macroeconomic (TRYM) Model of the Australian Economy*, Australian Treasury, Canberra.

Treasury 1981, The NIF-10 Model of the Australian Economy, AGPS, Canberra.

Treasury 1984, Proceedings of the Conference on the NIF-10 model, AGPS, Canberra.

Turnovsky, S 2002, 'Knife-Edge Conditions and the Macrodynamics of Small Open Economies', *Macroeconomic Dynamics*, 6, pp. 307–335.

Uzawa, H 1961, "Neutral Inventions and the Stability of Growth Equilibrium," *Review of Economic Studies,* Vol. 28 (2): pp. 117-124.

Wren-Lewis, S, 2018, 'Ending the microfoundations hegemony', Oxford Review of Economic Policy, 34, pp. 55-69.

Appendix A: Key equations

Equation 1: Labour force participation rate (cyclical component)

$$\Delta \log(\rho_t) = \beta_1[\log(\rho_{t-1}) - \log(\rho^*_{t-1})] + \beta_2\left[\log\left(\frac{N_t}{POP_t}\right) - \log\left(\frac{N^*_t}{POP^*_t}\right)\right] + \beta_3\Delta\log\left(\frac{N_{t-1}}{POP_{t-1}}\right) \tag{1}$$

Equation 2: Household consumption

$$\Delta\log(C_t) = \beta_6[\log(C_{t-1}) - \log(C^*_{t-1})] + \beta_7\Delta\log(Y^{NP}_t) + (1-\beta_7)\Delta\log(Y^*_t) + \beta_8\Delta U_t \tag{2}$$

Equilibrium household consumption

$$\log(C^*_t) = \beta_1 + \beta_2 dum\_c_t + \log(Y^{NP}_t) + \beta_3\left(\frac{VW^H_{t-1}}{Y^{NP}_t}\right) + \beta_4\left(\frac{VW^{NH}_{t-1}}{Y^{NP}_t}\right) + \beta_5 R_t \tag{2a}$$

Equation 3: Business investment rate in industry i (disequilibrium component)

$$IC_{i,t} = \beta_1\left(AR_{i,t-1} - CC_{i,t-1}\right) + \beta_2 IC_{i,t-1} \tag{3}$$

Definition of disequilibrium component of business investment rate

$$IC_{i,t} \equiv \frac{I_{i,t}}{K_{i,t-1}} - \left(\Delta\log(Y^*_t) + \frac{\delta_{i,t}}{400}\right) \tag{3a}$$

Actual rate of return on capital from cash flow

$$AR_{i,t} = \frac{(1-\tau^C_{i,t})\left[P^V_{i,t}\cdot V_{i,t} - \left(W_{i,t}\cdot NH_{i,t} + r^F_{i,t}\cdot F_{i,t}\right)\right]}{P^I_{i,t}\cdot K_{i,t}} + \tau^C_{i,t}\frac{\delta_{i,t}}{400} \tag{3b}$$

Cost of capital

$$CC_{i,t} = \frac{(i^{10}_t - \pi^e_t)}{400} + \left(1 + \frac{\pi^e_t}{400}\right)\frac{\delta_{i,t}}{400} + Risk_{i,t} \tag{3c}$$

Equation 4: Housing investment rate (disequilibrium component)

$$IC_{odw,t} = \beta_1\left(AR_{odw,t} - CC_{odw,t}\right) + \beta_2 IC_{odw,t-1} + \beta_3\Delta_4(i_{t-1} - i^{10}_{t-1}) \tag{4}$$

Definition of disequilibrium component of housing investment rate

$$IC_{odw,t} \equiv \frac{I_{odw,t}}{K_{odw,t-1}} - \left(\Delta y^*_t + \frac{\delta_{odw,t}}{400}\right) \tag{4a}$$

Actual rate of return on capital

$$AR_{odw,t} = \frac{P^V_{odw,t} \cdot V_{odw,t} - r^F_{odw,t} \cdot F_{odw,t}}{P^I_{odw,t} \cdot K_{odw,t-1}} \quad (4b)$$

Cost of capital

$$CC_{odw,t} = \frac{(i^{10}_t - \pi^e_t)}{400} + \left(1 + \frac{\pi^e_t}{400}\right)\frac{\delta_{odw,t}}{400} + Risk_{odw,t} \quad (4c)$$

Equation 5: Price of local sales of industry in the non-commodities industry

$$\Delta\log(P^E_{NC,t}) = \beta_1\left[\log(P^E_{NC,t-1}) - \log(P^{E^*}_{i,t-1})\right] + \beta_2\Delta\log(P^M_{NC,t-1}) + \beta_3\Delta\log(P^{E^*}_{NC,t}) + (1 - \beta_2 - \beta_3)\pi^T_t \quad (5)$$

Equilibrium price of local sales, P^{E^*}, of industry *i* is determined recursively in a series of four equations as follows.

Equilibrium price of gross value added, P^{V^*}, is equal to its marginal cost of production, satisfying the marginal product of labour condition.

$$P^{V^*}_{NC,t} = \left(\frac{W_t}{\lambda^N_{NC,t}}\right)\left[\frac{1 - \theta^{K\,\frac{1}{\sigma^V_i}}_{NC}\left(\frac{K_{i,t-1}}{V^*_{i,t}}\right)^{\frac{\sigma^V_i - 1}{\sigma^V_i}}}{\theta^N_{NC}}\right]^{\frac{1}{\sigma^V_i - 1}} \quad (5a)$$

Equilibrium price of domestic production, P^{D^*}, is then determined as a weighted average of the equilibrium price of gross value added, P^{V^*} and intermediate input prices.

$$P^{D^*}_{i,t} = \beta_v\, P^{V^*}_{i,t} + \sum_j \beta_{j,i} P^E_{j,t} \quad (5b)$$

Equilibrium price of total supply, P^{Y^*}, is then determined as a CES cost function in the equilibrium price of domestic production, P^{D^*}, and import prices, P^M.

$$P^{Y^*}_{i,t} = \left[\alpha_i {P^{D^*}_{i,t}}^{1-\sigma^Y_i} + (1 - \alpha_i){P^M_{i,t}}^{1-\sigma^Y_i}\right]^{\frac{1}{1-\sigma^Y_i}} \quad (5c)$$

Finally, the equilibrium price of local sales, P^{E^*}, is determined residually from a CET revenue function in which the equilibrium price of total supply, P^{Y^*}, reflects the equilibrium price of local sales, P^E, and export prices, P^X.

$$P^{Y^*}_{i,t} = \left[\phi_i\left(P^{X^*}_{i,t}\right)^{1+\sigma^T_i} + (1 - \phi_i)\left(P^{E^*}_{i,t}\right)^{1+\sigma^T_i}\right]^{\frac{1}{1+\sigma^T_i}} \quad (5d)$$

Equation 6: Total hours worked in industry i

$$\Delta \log (NH_{i,t}) = \beta_1 \left[(NH_{i,t-1}) - \log (NH^*_{i,t-1}) \right] + \beta_2 \Delta \log (NH^*_{i,t}) + (1-\beta_2)(\Delta \log (POP^*_t) + \Delta \log (H^*_t) + \Delta \log (\rho^*_t)) \quad (6)$$

Equilibrium total hours worked (by inverting this CES production function)

$$V^*_{i,t} = A_{i,t} \left[\theta_i^{N \frac{1}{\sigma_i^V}} \left(\lambda^N_{i,t} NH^*_{i,t} \right)^{\frac{\sigma_i^V - 1}{\sigma_i^V}} + \theta_i^{K \frac{1}{\sigma_i^V}} K_{i,t-1}^{\frac{\sigma_i^V - 1}{\sigma_i^V}} \right]^{\frac{\sigma_i^V}{\sigma_i^V - 1}} \quad (6a)$$

Equation 7: Average hours worked in industry i

$$\log (H^c_{i,t}) = \beta_1 \Delta \log (NH^c_{i,t}) + \beta_2 \log (H^c_{i,t-1}) \quad (7)$$

Cyclical average hours worked in industry i

$$\log (H^c_{i,t}) \equiv \log (H_{i,t}) - \log (H^*_{i,t}) \quad (7a)$$

Cyclical total hours worked in industry i

$$\log (NH^c_{i,t}) \equiv \log (NH_{i,t}) - \log (NH^*_{i,t}) \quad (7b)$$

Equation 8: Imports that compete with industry i

$$\Delta \log (\mathrm{M}) = \beta_1 \left[\log (M_{i,t-1}) - \log (M^*_{i,t-1}) \right] + \beta_2 \Delta \log (Y_{i,t}) + \beta_3 \left(\Delta \log \left(P^M_{i,t} \right) - \Delta \log \left(P^Y_{i,t} \right) \right) + (1-\beta_2) \Delta \log (Y^*_t) \quad (8)$$

Equilibrium imports in competition with industry i (based on cost minimising combination of imports, M, and domestic production, D, in producing total supply, Y)

$$M^*_{i,t} = (1-\alpha_i) Y_{i,t} \left(\frac{P^{Y*}_{i,t}}{P^M_{i,t}} \right)^{\sigma_i^Y} \quad (8a)$$

Equation 9: Price of exports for the non-commodities industry

$$\Delta \log (P^X_{nc,t}) = \beta_1 \left(\log (P^X_{nc,t-1}) - \beta_2 P^X_{nc,t} trend - \log (P^{X*}_{nc,t-1}) \right) + \beta_3 \left(\Delta \log \left(\frac{P^W_t}{\varepsilon_t} \right) \right) + (1-\beta_3) \Delta \log ({P^X_{nc,t}}^*) \quad (9)$$

Equilibrium price of exports for the non-commodities industry (based on revenue-maximising combination of exports and domestic sales from total supply).

$${P^X_{nc,t}}^* = {P^E_{nc,t}}^* \left[\frac{1-\phi_{nc}}{\phi_{nc}} \frac{X_{nc,t}}{E_{nc,t}} \right]^{\frac{1}{\sigma^T_{nc}}} \quad (9a)$$

Equation 10: Inventory investment

$$\Delta\log(KST_t) = \beta_3[\log(KST_{t-1}) - \log(KST^*_{t-1})] + \beta_4\Delta log(KST_{t-1}) + (1-\beta_4)\Delta\log(Y^*_t) \tag{10}$$

Equilibrium stock of inventories

$$\log(KST^*_t) = \beta_1 + \beta_2 trend_t + \log(Y_t) \tag{10a}$$

Equation 11: Price of imports of industry i

$$\Delta\log(P^M_{i,t}) = \beta_1\left[\log\left(P^M_{i,t-1}\right) - \beta_2 - \beta_3 trend_{t-1} - \log\left({P^M_{i,t-1}}^*\right)\right] + \beta_4\Delta\log(\varepsilon_t) + \beta_5\Delta\log(P^F_{i,t}) + (1-\beta_5)\Delta\log(P^F_{i,t-1}) \tag{11}$$

Equilibrium price of imports that compete with industry i

$${P^M_{i,t}}^* = \frac{P^F_{i,t}}{\varepsilon_t} \tag{11a}$$

Equation 12: Export demand for non-commodities

$$\Delta\log\left(X_{NC,t}\right) = \beta_1\left[\log\left(X_{NC,t-1}\right) - \delta_t - \log(X^*_{nc,t-1})\right] + \beta_3\Delta\left[\log(P^X_{NC,t}/P^F_{NC,t}) \cdot \varepsilon_t\right] + \beta_4\Delta\log(Y^F_t) + (1-\beta_4)\Delta\log(Y^*_t) \tag{12}$$

Equilibrium exports of industry i

$$X^*_{i,t} = Y^F_t\left(\frac{{P^X_{i,t}}^*\varepsilon_t}{P^F_{i,t}}\right)^{\epsilon} \tag{12a}$$

Equation 13: General government consumption (exclusive of depreciation)

$$\Delta\log(G^c_t) = \beta_2(\log(G^c_{t-1}) - \log({G^c_{t-1}}^*)) + \Delta\log(Y^*_t) \tag{13}$$

Equilibrium government consumption (exclusive of depreciation)

$$\log({G^c_t}^*) \equiv \beta_1 + \log(Y_t) \tag{13a}$$

Equation 14: General government investment

$$\Delta\log(G^i_t) = \beta_2\left(\log(G^i_{t-1}) - \log({G^i_{t-1}}^*)\right) + \Delta\log(Y^*_t) \tag{14}$$

Equilibrium government investment (exclusive of depreciation)

$$\log({G^i_t}^*) \equiv \beta_1 + \log(Y_t) \tag{14a}$$

Equation 15: Fiscal policy rule

$$\Delta\tau_t^{HH} = -0.007\left(\frac{B_t}{YZ_t{}^*} - B_t^*\right) - 0.12\,\Delta\left(\frac{B_t}{YZ_t{}^*} - B_t^*\right) \tag{15}$$

Equation 16: Monetary policy rule for cash rate

$$\Delta i_t = 0.3\,(i_t^* - i_{t-1}) - \Delta_2(U_t - U_t^*) \tag{16}$$

$$i_t^* = r_t^* + \pi_t^{TM} + (\pi_t^{TM} - \pi_t^*) - 2\,(U_t - U_t^*) \tag{16a}$$

Equation 17: Wage equation

Signal equation:

$$\begin{aligned}\Delta\log(W_t) - \Delta\log(Z_t) - \pi_t^{TM} &= \beta_1(\pi_t^e - \pi_{t-1}^{TM}) + D_{<1993}\beta_2\left(\frac{U_t - U_t^*}{U_t}\right) + D_{\geq 1993}\beta_3\left(\frac{U_t - U_t^*}{U_t}\right) \\ &+ \beta_4\left(\frac{\Delta U_{t-1}}{U_t}\right) + \beta_5(\Delta\log(\mathrm{Z}_t^*) - \Delta\log(\mathrm{Z}_t)) + v_t\end{aligned} \tag{17}$$

State equation:

$$U_t^* = U_{t-1}^* + \xi_t$$

Household expected inflation (percentage per year)

$$\Delta\pi_t^e = \beta_1(\pi_{t-1}^e - \pi_t^*) + \beta_2\,(\pi_{t-1}^{TM} - \pi_{t-1}^e) \tag{17a}$$

Equation 18: Exchange rate

$$log(\varepsilon_t) = log(\varepsilon_{t+1,t}) + \ log\left(1 + \frac{i_t}{400}\right) - log\left(1 + \frac{i_t^F}{400}\right) - risk\_\varepsilon_t \tag{18}$$

Equation 19: 10-year bond rate

$$i_t^{10} = risk\_i10_t + (1 - 0.95)i_t + 0.95(i_{t+1,t}^{10} - risk\_i10_{t+1,t}) \tag{19}$$

Equation 20: 10-year expected inflation

$$\pi_t{}^{e,10} = (1 - 0.95)\pi_t + 0.95\,\pi_{t+1,t}^{e,10} \tag{20}$$

Appendix B: Derivation of Business Investment Equation

This attachment derives the business investment equation used in EMMA's three industry sectors. More a more general discussion of business investment in EMMA, see the section of the main body of this chapter that discusses the EMMA approach to the business sector.

In the tradition of the Tobin-q theory of investment, a representative firm in each industry maximises the present value of its cash flow in the presence of capital stock adjustment costs, to account for the sluggish adjustment of the capital stock. The standard approach to the Tobin-q theory of investment that uses capital and labour as its primary factors of production has been extended to include a fixed factor of production as a third primary factor.

The theoretical derivation of the investment equation is given in the next section, while the final section develops this into the final equation used in the model.

Theory equation

The representative business in an industry produces output (V) using inputs of capital (K), labour[16] (N) and a fixed factor (F). It chooses a plan for its employment (N), the fixed factor (F) and investment (I) that maximises the present value of its after-tax cash flow. After-tax cash flow is calculated as cash flow ($CFBT$) net of company tax (TB). In calculating present values, the expected real quarterly discount rate (r) is able to vary from quarter-to-quarter.

Under Tobin's-q theory of investment, adjustment costs account for the sluggish adjustment of the capital stock. In the version of adjustment costs used here, it is assumed they are incurred only during the transition path from one steady state to another, as in Kudrna & Woodland (2011). In particular, adjustment costs depend on the deviation of investment (I) from its steady state level ($(\delta + gr)K$). The nature of these adjustment costs is assumed to be such that they are not an allowable deduction for company tax.

$$\mathcal{L} = \sum_{u=t}^{\infty} \frac{1}{\prod_{i=0}^{u-t}(1 + r_{t+i})} \{CFBT_u - TB_u + \mu_{u+1} [(1 - \delta) K_u + I_u - K_{u+1}]\}$$

$$CFBT_u = PV_u\, f(K_u, F_u, N_u) - W_u\, N_u - R_u\, F_u - PI_u\, I_u - \frac{\psi}{2} PI_u \frac{[I_u - (\delta + gr)\, K_u]^2}{K_u}$$

$$TB_u = rtb_u\, [PV_u\, (K_u, F_u, N_u) - W_u\, N_u - R_u\, F_u - \delta\, PI_u\, K_u]$$

16 N here refers to total labour inputs, NH elsewhere.

Maximising this Lagrangian yields four first order conditions for employment, the fixed factor, investment and the physical capital stock, as well as the capital accumulation constraint. After simplifying, these four first order conditions are as follows.

$$PV_t \frac{\partial f}{\partial N_t} = W_t \tag{A1.1}$$

$$PV_t \frac{\partial f}{\partial F_t} = R_t \tag{A1.2}$$

$$\frac{I_t}{K_t} = \delta + gr + \frac{1}{\psi}\left[\frac{\mu_{t+1}}{PI_t} - 1\right] \tag{A1.3}$$

$$\mu_t = \left\{ \begin{array}{c} (1-\delta)\,\mu_{t+1} + (1-rtb_t)\,PV_t\,\frac{\partial f}{\partial K_t} \\ +\,\psi\,(\delta+gr)\,PI_t\left[\frac{I_t}{K_t} - (\delta+gr)\right] \\ +\frac{\psi}{2}\,PI_t\left[\frac{I_t}{K_t} - (\delta+gr)\right]^2 + rtb_t\,\delta\,PI_t \end{array} \right\} \Big/ (1+r_t) \tag{A1.4}$$

Equation (A1.1) is the marginal product of labour condition, while equation (A1.2) is the marginal product of fixed factor condition. Equation (A1.3) is Tobin's-q theory of investment under which the rate of investment depends on the ratio of the market value of a unit of capital to its replacement cost. Equation (A1.4) is forward-looking, determining the evolution over time of the market price of a unit of capital.

From these foundations, there are a range of options for developing an investment equation. The first option is to assume that investors' expectations for the market price of a unit of capital are model consistent. In that case, those expectations are obtained using equation (A1.4). Those expectations are in turn used in the q investment equation (A1.3).

Another option is to assume that investors' expectations for the market price of a unit of capital are static. The general idea behind static expectations is that investors are not sophisticated enough to understand that when they lift investment in response to a high q-ratio, other investors are likely to do the same thing, increasing the supply of capital and hence pushing down the q-ratio. EMMA assumes static expectations.

Applying static expectations to equation (A1.4) and solving for the q-ratio gives equation (A1.5).

$$q_t = \frac{\mu_t}{PI_t} = \left\{(1-rtb_t)\,\frac{PV_t}{PI_t}\,\frac{\partial f}{\partial K_t} + \psi\,(\delta+gr)\,\tilde{I}_t + \frac{\psi}{2}\tilde{I}_t^2 + rtb_t\,\delta\right\} \Big/ (\delta + r_t) \tag{A1.5}$$

This equation has been simplified by defining a new variable for the deviation of the investment rate from its steady-state value (that is, disequilibrium investment).

$$\tilde{I}_t = \frac{I_t}{K_t} - (\delta + gr) \tag{A1.6}$$

Using the same simplifications, the investment equation (equation (A1.3)) can be re-written more compactly as equation (A1.7).

$$\tilde{I}_t = \frac{1}{\psi}\left[q_t - 1\right] \quad \text{(A1.7)}$$

Using equations (A1.5) and (A1.7) to eliminate the q-ratio gives a quadratic equation in disequilibrium investment.

$$\frac{\psi}{2}\tilde{I}_t^2 - \psi\,(r_t - gr)\,\tilde{I}_t + (1 - rtb_t)\,\frac{PV_t}{PI_t}\,\frac{\partial f}{\partial K_t} + rtb_t\,\delta - [\delta + r_t] = 0 \quad \text{(A1.8)}$$

The constant term in this quadratic equation is the difference, expressed in after-tax terms, between the actual and required rates of return on a marginal unit of capital.

$$\frac{\psi}{2}\tilde{I}_t^2 - \psi\,(r_t - gr)\,\tilde{I}_t + ARAT_t - RRAT_t = 0 \quad \text{(A1.9)}$$

where:

$$ARAT_t = (1 - rtb_t)\,\frac{PV_t}{PI_t}\,\frac{\partial f}{\partial K_t} + rtb_t\,\delta \quad \text{(A1.10)}$$

$$RRAT_t = \delta + r_t \quad \text{(A1.11)}$$

Technically, this quadratic equation has two solutions for disequilibrium investment. We select the solution consistent with the attainment of an equilibrium in which disequilibrium investment is zero when $ARAT = RRAT$.

$$\tilde{I}_t = (r_t - gr) - \sqrt{(r_t - gr)^2 - \frac{2}{\psi}\,(ARAT_t - RRAT_t)} \quad \text{(A1.12)}$$

Equation (A1.12) could be used in EMMA as a non-linear investment equation. However, it is more convenient to linearise it around the steady state solution, which gives the simpler equation (A1.13).

$$\tilde{I}_t = \frac{1}{\psi.(r_0 - gr)}\,(ARAT_t - RRAT_t) \quad \text{(A1.13)}$$

Final equation

It remains to use equation (A1.13) to develop a final equation used in EMMA. First, we treat the coefficient on the difference between the after-tax actual and required rates of return as an estimation parameter, β_1.

$$\tilde{I}_t = \beta_1 \left(ARAT_t - RRAT_t\right) \qquad \text{(A1.14)}$$

Second, in deriving the above equation, we took account of costs of adjusting the capital stock, but not the costs of adjusting investment. The effect of taking into account costs of adjusting the capital stock was that the capital stock adjusts gradually, guided by differences between the actual and required rates of return, as seen in equation (A1.14).

Groth and Kahn (2010) find that costs of adjusting investment are empirically important and lead to sluggish adjustment of investment i.e., investment depending on its own lagged value. This idea can be captured by superimposing on equation (A1.14) partial adjustment of the disequilibrium investment rate, as seen in equation (A1.15).

$$\tilde{I}_t = \beta_1 \left(ARAT_t - RRAT_t\right) + \beta_2\, \tilde{I}_{t-1} \qquad \text{(A1.15)}$$

Third, we re-express the after-tax actual rate of return, ARAT, so that it is more closely connected to cash flow, because of the intuitive appeal of relating investment to cash flow. Under constant returns to scale, Euler's theorem implies the following.

$$V_t = \frac{\partial f}{\partial K_t}\, K_t + \frac{\partial f}{\partial F_t}\, F_t + \frac{\partial f}{\partial N}\, N_t$$

We can use this to eliminate the marginal product of capital in the equation for the after-tax actual rate of return, equation (A1.10). While this introduces the marginal products for labour and the fixed factor, these are eliminated using equations (A1.1) and (A1.2) respectively.

$$ARAT_t = \frac{(1 - rtb_t)\left[PV_t\, V_t - W_t\, N_t - RF_t\, F_t\right]}{PI_t\, K_t} + rtb_t\, \delta \qquad \text{(A1.16)}$$

This shows that the incentive to invest depends on revenue net of both labour costs and economic rents. Here economic rents are the return to the fixed factor.

In calculating the actual rate of return on a post-tax basis, taxation expenses are calculated by applying the tax rate (rtb) to profits, and then adding back the value of the tax deduction for depreciation, which is the final term in equation (A1.16).

Fourth, in discounting future cash flows using the required rate of return, equation (A1.11) is unrealistic in implicitly assuming static expectations with respect to the discount rates that link each quarter. This is because there are well-developed bond markets in which the term structure of interest rates, and hence future expected short-term interest rates, are readily observable. Hence, in defining the required rate of return, the real discount rates used should not just refer to one quarter ahead, but rather should reflect the typical time horizon of investment decisions. This occurs automatically if model-consistent expectations are assumed. Under static expectations, it can be done in an approximate way by using a long-term interest rate. The real discount rate should also allow for risk.

Taking these considerations into account, equation (A1.17) for the required rate of return uses as the real discount rate a nominal bond rate ($RI10$) net of expected inflation over the same time horizon (π_t^e) plus an allowance for risk ($QRISK$). Annual, percentage rates are converted to quarterly, proportionate rates by dividing by 400.

$$RRAT_t = \delta + \frac{RI10_t - \pi_t^e}{400} + QRISK_{i,t} \qquad \text{(A1.17)}$$

This means that the investment equation for each industry is given by equation (A1.15), where disequilibrium investment is defined by equation (A1.6), the after-tax actual rate of return, $ARAT$, by equation (A1.16) and the after-tax required rate of return, $RRAT$, by equation (A1.17).

This same set of equations appears in the list of key equations in Attachment A, after conversion to EMMA model notation and units of measurement. Specifically, equations (A1.15), (A1.6), (A1.16) and (A1.17) from here, appear there as equations (3), (3a), (3b) and (3c) respectively.

Appendix C: Productivity

This attachment describes the approach to modelling productivity in EMMA.

Firm value-added production in each industry is explained by a constant elasticity of substitution (CES) production function with constant returns to scale. This production function explains how firms combine factor inputs of capital (K), labour (NH) and a fixed factor (F) to produce value added output (V).

$$V_{i,t} = A_{i,t}\left[\theta_i^{N\frac{1}{\sigma_i^V}}\left(\lambda_{i,t}^N NH_{i,t}\right)^{\frac{\sigma_i^V-1}{\sigma_i^V}} + \theta_i^{K\frac{1}{\sigma_i^V}}\left(\xi_{i,t}K_{i,t-1}\right)^{\frac{\sigma_i^V-1}{\sigma_i^V}} + \theta_i^{F\frac{1}{\sigma_i^V}}\left(\lambda_{i,t}^N F_{i,t}\right)^{\frac{\sigma_i^V-1}{\sigma_i^V}}\right]^{\frac{\sigma_i^V}{\sigma_i^V-1}} \quad \text{(A2.1)}$$

Where, for each industry i: $V_{i,t}$ is GVA at basic prices; $A_{i,t}$ is total factor productivity; θ's represent each factor's respective contribution to output such that $\theta_i^N + \theta_i^K + \theta_i^F = 1$; σ_i^V is the elasticity of substitution between the factors; $\lambda_{i,t}^N$ represents the level of labour-augmenting technical change; $NH_{i,t}$is total hours worked; $\xi_{i,t}$ is capacity utilisation; $K_{i,t-1}$ is the capital stock; and, $F_{i,t}$ is the fixed factor input.

Labour productivity reflects the ability of labour to transform its input of hours worked into value-added production. Labour productivity can be derived by re-arranging the CES production function in Equation A2.1 (shown for the non-commodities industry which does not include a fixed factor):

$$\frac{V_{i,t}}{NH_{i,t}} = \left(A_{i,t}\lambda_{i,t}^N\right)^{\frac{\sigma_i^V}{\sigma_i^V-1}}\left[\theta_i^{N\frac{1}{\sigma_i^V}} + \left(1-\theta_i^N\right)^{\frac{1}{\sigma_i^V}}\left(\frac{\xi_{i,t}K_{i,t-1}}{\lambda_{i,t}^N NH_{i,t}}\right)^{\frac{\sigma_i^V-1}{\sigma_i^V}}\right]^{\frac{\sigma_i^V}{\sigma_i^V-1}} \quad \text{(A2.2)}$$

Taking difference logs, it can be shown that labour productivity growth is equal to the sum of:

- the growth rate of total factor productivity ($A_{i,t}$) (Hicks neutral productivity);
- the growth rate of labour-augmenting technical change ($\lambda_{i,t}^N$) (Harrod neutral productivity); and
- the rate of capital deepening (capital per effective unit of labour ($\frac{\xi i_{,t} K_{t-1}}{\lambda_{i,t}^N NHi_{,t}}$))

$$\Delta\log\left(\frac{Y_{i,t}}{NH_{i,t}}\right) = \Delta\log\left(\lambda_{i,t}^N\right) + \Delta\log\left(A_{i,t}\right) + \Delta\log\left(\frac{\xi_{i,t}K_{i,t-1}}{\lambda_{i,t}^N NH_t}\right) \quad \text{(A2.3)}$$

Total factor productivity (Hicks neutral) is defined as improvements in productivity that have a symmetrical impact on the productivity of labour and capital (and the fixed factor in the case of the commodity industries). That is, an increase in total factor productivity will increase the marginal productivity of labour (MPL) and capital (MPK) by the same percentage.

Harrod neutral productivity means that improvements in productivity are specific to a factor of production. This is often called labour or capital-augmenting technical change. Improvements in the productive capacity of capital are embodied in the value of the capital stock itself rather than augmenting the capital stock.[17] The focus is therefore on labour augmenting technical change.

Balanced growth refers to an allocation where output grows at a constant rate and the capital-output ratio, the interest rate and factor shares remain constant. These are the so-called Kaldor facts (Kaldor, 1963). Balanced growth is a desirable property for a macroeconometric model like EMMA since it ensures the long-run stability of the model. The Kaldor facts are broadly consistent with Australian data. Notably, after accounting for changes in industry composition, factor income shares have remained broadly constant over the past 30 years.

Uzawa's theorem (Uzawa, 1961) shows that constant growth of output, capital and consumption combined with constant returns to scale implies that the aggregate production function must have a representation with Harrod-neutral (purely labour-augmenting) technological progress. The intuition of this result follows from the fact that capital accumulates whereas labour supply is exogenous. That is, increases in labour-augmenting technical change can induce increases in the productive capital stock and hence allows for factor inputs shares to be maintained, whereas increases in capital-augmenting technical change do not induce increases in labour supply.[18]

Labour augmenting technical change and total factor productivity cannot be distinguished in the labour demand equation without additional identifying assumptions. Following the business cycle literature, total factor productivity has been modelled as a first-order autoregressive stochastic process.

Total factor productivity and capital deepening are assumed to be cyclical, and are therefore stationary in the long run. This means they contribute to productivity growth in the short run, but in equilibrium they are constant and so labour productivity grows in line with labour-augmenting technical change.

Labour-augmenting technical change, total factor productivity and capital utilisation are unobserved. The first order condition on labour, combined with data on the real producer wage, has been used to extract the unobserved labour-augmenting technical change and total factor productivity components. It is then assumed that capital utilisation captures the remaining variation in observed value-added output.

The first order condition for labour from the firm's profit maximisation problem sets the marginal product of labour equal to the real producer wage ($\frac{W_t}{P_{i,t}}$):

$$\frac{\partial Y_{i,t}}{\partial NH_{i,t}} = \left(\lambda_{i,t}^N A_{i,t}\right)^{\frac{\sigma_i^V - 1}{\sigma_i^V}} \theta_i^{N\frac{1}{\sigma_i^V}} \left(\frac{Y_{i,t}}{NH_{i,t}}\right)^{\frac{1}{\sigma_i^V}} = \frac{W_t}{P_{i,t}}$$

17 Cyclical variations in the utilisation of capital are captured through the term $\xi_{i,t}$.

18 The Cobb-Douglas production function is a special case under which it is not possible to separately identify Hicks and Harrod neutral productivity. The Cobb-Douglas production function can be represented as purely labour-augmenting and so total factor productivity in a Cobb-Douglas production function is consistent with balanced growth.

We rearrange the first order condition to give the following equilibrium labour demand equation:

$$NH_{i,t} = \left(A_{i,t}\lambda_{i,t}^{N}\right)^{\sigma_i^V - 1}\theta_i^N Y_{i,t}\left(\frac{W_t}{P_{i,t}}\right)^{-\sigma_i^V} \tag{A2.4}$$

This long-run relationship can be log-linearised and fitted to data.

Given the assumption that total factor productivity is stationary and mean-zero, it will capture deviations in the long-run relationship between employment and equilibrium employment (nh_t^*) as derived from the first order condition:

$$nh_{i,t}^* = (\sigma_i^V - 1)\lambda_{i,t}^n + \theta_i^N + y_{i,t} - \sigma_i^V\left[w_t - p_{i,t}\right] \tag{A2.5}$$

$$a_{i,t} = nh_{i,t} - nh_{i,t}^* \tag{A2.6}$$

Substituting in our assumption of cyclical total factor productivity gives our signal equation for identifying labour-augmenting technical change:

$$\begin{gathered}\Delta nh_{i,t} = \Delta nh_{i,t}^* + (\rho - 1)\left[nh_{i,t-1} - nh_{i,t-1}^*\right] + (\sigma_i^V - 1)v_t \\ v_t \sim (0, \sigma_v^2)\end{gathered} \tag{A2.7}$$

We model the unobserved component of labour-augmenting technical change as a stochastic trend, in particular a random walk with time-varying drift (δ_t). The time-varying drift is the second state equation in the system.

Signal equation:

$$\begin{aligned}\Delta nh_{i,t} = \Delta&\left\{(\sigma_i^V - 1)\lambda_{i,t}^N + \theta_i^N + y_{i,t} - \sigma_i^V\left[w_t - p_{i,t}\right]\right\} \\ &+ (\rho - 1)\left[nh_{i,t-1} - \left\{(\sigma_i^V - 1)\lambda_{i,t-1}^N + \theta_i^N + y_{i,t-1} - \sigma_i^V\left[w_{t-1} - p_{i,t-1}\right]\right\}\right] \\ &+ (\sigma_i^V - 1)v_t \\ &\qquad v_t \sim (0, \sigma_v^2)\end{aligned} \tag{A2.8}$$

State equations:

$$\lambda_{i,t}^N = \delta_t + \lambda_{i,t-1}^N \tag{A2.9}$$

$$\delta_t = \delta_{t-1} + \epsilon_t \tag{A2.10}$$

$$\epsilon_t \sim (0, \sigma_\epsilon^2)$$

Capital utilisation has been identified as the residual of the production function:

$$\xi_{i,t} = \frac{Y_{i,t}}{A_{i,t}K_{i,t-1}} \frac{1}{(1-\theta_i^N)^{\frac{1}{\sigma_i^V-1}}} \left[1-\theta_i^{N\frac{1}{\sigma^V}} \left(\frac{A_{i,t}\lambda_{i,t}^N \cdot N_{i,t}}{Y_{i,t}}\right)^{\frac{\sigma_i^V-1}{\sigma_i^V}}\right]^{\frac{\sigma_i^V}{\sigma_i^V-1}} \quad \text{(A2.11)}$$

This identification strategy means that a change in total factor productivity or capital utilisation has a temporary effect on the level of measured labour productivity, while shocks to labour augmenting technical change have a permanent effect.

Appendix D: Business Sector Equilibrium equations

In EMMA, in the long run, a single profit maximisation problem explains the behaviour of firms and ensures internal consistency between the firm's input demand and supply decisions. Different functional forms are used for non-commodities sector and the trade-orientated commodities sectors of mining and agriculture. The two commodity industries are assumed to be Classical, so that prices are flexible and producers operate on their supply curves. The non-commodity industry is assumed to be Keynesian, so that its price is sticky and output is demand determined in the short run.

Variable	Non-Commodities	Commodities
Imports ($\boldsymbol{M_{i,t}}$)	$M^*_{i,t} = (1-\alpha_i)Y_{i,t}\left(\frac{P^{Y\,*}_{i,t}}{P^{M\,*}_{i,t}}\right)^{\sigma_i^Y}$	$M^*_{i,t} = \left(\frac{\alpha_i}{1-\alpha_i}\right)^{-1} D_{i,t}\left(\frac{P^D_{i,t}}{P^M_{i,t}}\right)^{\sigma_i^Y}$
Exports ($\boldsymbol{X_{i,t}}$)	$X^*_{i,t} = Y^F_{i,t}\left(\frac{P^{X\,*}_{i,t}\varepsilon_t}{P^F_{i,t}}\right)^{\epsilon}$	$X^*_{i,t} = \phi_i^{\frac{1}{\sigma_i^T+1}}\left[Y^*_{i,t}{}^{\frac{\sigma_i^T+1}{\sigma_i^T}} - (1-\phi_i)^{\frac{-1}{\sigma_i^T}} E_{i,t}{}^{\frac{\sigma_i^T+1}{\sigma_i^T}}\right]^{\frac{\sigma_i^T}{\sigma_i^T+1}}$
Domestic supply ($\boldsymbol{E_{i,t}}$)	Demand driven	Demand driven
Total supply ($\boldsymbol{Y_{i,t}}$)	Demand driven	$Y^*_{i,t} = \left[\alpha_i^{\frac{1}{\sigma_i^Y}}\left(D^*_{i,t}\right)^{\frac{\sigma_i^Y-1}{\sigma_i^Y}} + (1-\alpha_i)^{\frac{1}{\sigma_i^Y}}\left(M^*_{i,t}\right)^{\frac{\sigma_i^Y-1}{\sigma_i^Y}}\right]^{\frac{\sigma_i^Y}{\sigma_i^Y-1}}$
Domestic production ($\boldsymbol{D_{i,t}}$)	$D^*_{i,t} = \alpha_i^{\frac{1}{1-\sigma_i^Y}}\left[Y_{i,t}{}^{\frac{\sigma_i^Y-1}{\sigma_i^Y}} - (1-\alpha_i)^{\frac{1}{\sigma_i^Y}} M^*_{i,t}{}^{\frac{\sigma_i^Y-1}{\sigma_i^Y}}\right]^{\frac{\sigma_i^Y}{\sigma_i^Y-1}}$	$D^*_{i,t} = \frac{V^*_{i,t}}{(1-\sum_i \beta_i)}$

Variable	Non-Commodities	Commodities
Value added ($\boldsymbol{V_{i,t}}$)	$V^*_{i,t} = D^*_{i,t}\left(1 - \sum_i \beta_i\right)$	$V^*_{i,t} = \dfrac{A_{i,t}\left[\theta_i^{K\frac{1}{\sigma_i^V}} K_{i,t-1}^{\frac{\sigma_i^V-1}{\sigma_i^V}} + \theta_i^{F\frac{1}{\sigma_i^V}}\left(\lambda^N_{i,t}F_{i,t}\right)^{\frac{\sigma_i^V-1}{\sigma_i^V}}\right]^{\frac{\sigma_i^V}{\sigma_i^V-1}}}{\left[1 - \theta_i^N\left(A_{i,t}\lambda^N_{i,t}\right)^{\sigma_i^V-1}\left(\frac{W_t}{P^V_{i,t}}\right)^{1-\sigma_i^V}\right]^{\frac{\sigma_i^V}{\sigma_i^V-1}}}$
Import price ($\boldsymbol{P^M_{i,t}}$)	$P^{M\,*}_{i,t} = \dfrac{P^F_t}{\varepsilon_t}$	$P^{M\,*}_{i,t} = \dfrac{P^F_t}{\varepsilon_t}$
Export supply price ($\boldsymbol{P^X_{i,t}}$)	$P^{X\,*}_{i,t} = P^{E\,*}_{i,t}\left[\dfrac{X_{i,t}}{E_{i,t}} \cdot \dfrac{1-\phi_i}{\phi_i}\right]^{\frac{1}{\sigma_i^T}}$	$P^{X\,*}_{i,t} = \dfrac{P^W_{i,t}}{\varepsilon_t}\left(\dfrac{X_{i,t}}{Y^F_{i,t}}\right)^{\frac{-1}{\epsilon}}$
Domestic supply price ($\boldsymbol{P^E_{i,t}}$)	$P^{E\,*}_{i,t} = \left[\dfrac{1}{1-\phi_i} P^{Y\,*(1+\sigma_i^T)}_{i,t} - \dfrac{\phi_i}{1-\phi_i} P^{X\,*(1+\sigma_i^T)}_{i,t}\right]^{\frac{1}{1+\sigma_i^T}}$	$P^{E\,*}_{i,t} = P^{X\,*}_{i,t}\left[\dfrac{E_{i,t}}{X^*_{i,t}} \cdot \dfrac{\phi_i}{1-\phi_i}\right]^{\frac{1}{\sigma_i^T}}$
Total supply price ($\boldsymbol{P^Y_{i,t}}$)	$P^{Y\,*}_{i,t} = \left[\alpha_i \cdot P^{D\,*(1-\sigma_i^Y)}_{i,t} + (1-\alpha_i) \cdot P^{M(1-\sigma_i^Y)}_{i,t}\right]^{\frac{1}{1-\sigma_i^Y}}$	$P^{Y\,*}_{i,t} = \left[\phi_i \cdot P^{X\,*(1+\sigma_i^T)}_{i,t} + (1-\phi_i) \cdot P^{E\,*(1+\sigma_i^T)}_{i,t}\right]^{\frac{1}{1+\sigma_i^T}}$
Domestic production price ($\boldsymbol{P^D_{i,t}}$)	$P^{D\,*}_{i,t} = \left(1 - \sum_i \beta_i\right) P^{V\,*}_{i,t} + \sum_i \beta_i P^E_{i,t}$	$P^{D\,*}_{i,t} = \alpha_i^{\frac{1}{\sigma_i^Y-1}}\left[P^{Y\,*(1-\sigma_i^Y)}_{i,t} - (1-\alpha_i) \cdot P^{M*(1-\sigma_i^Y)}_{i,t}\right]^{\frac{1}{1-\sigma_i^Y}}$

Variable	Non-Commodities	Commodities
Value added price ($\boldsymbol{P_{i,t}^{V}}$)	$P_{i,t}^{V\,*} = \frac{1}{\theta_i^{N\frac{1}{\sigma_i^V}}} \frac{W_t}{A_{i,t}\lambda_{i,t}^N} \left[\frac{1}{\theta_i^{N\frac{1}{\sigma_i^V}}} - \frac{\theta_i^{K\frac{1}{\sigma_i^V}}}{\theta_i^{N\frac{1}{\sigma_i^V}}} \left(\frac{A_{i,t}K_{i,t-1}}{V_{i,t}} \right)^{\frac{\sigma_i^V-1}{\sigma_i^V}} \right]^{\frac{1}{\sigma_i^V-1}}$	$P_{i,t}^{V\,*} = \frac{\left[P_{i,t}^{D\,*} - \sum_i \beta_i P_{i,t}^E\right]}{\left(1 - \sum_i \beta_i\right)}$
Labour demand – total hours worked ($\boldsymbol{NH_{i,t}}$)	$NH_{i,t}^{*} = \left(\frac{1}{\lambda_{i,t}^N} \right) \theta_i^{N\frac{1}{1-\sigma_i^V}} \left[\left(\frac{V_{i,t}}{A_{i,t}} \right)^{\frac{\sigma_i^V-1}{\sigma_i^V}} - \theta_i^{K\frac{1}{\sigma_i^V}} K_{i,t-1}^{\frac{\sigma_i^V-1}{\sigma_i^V}} \right]^{\frac{\sigma_i^V}{\sigma_i^V-1}}$	$NH_{i,t}^{*} = \theta_i^N \left(A_{i,t}\lambda_{i,t}^N\right)^{\sigma_i^V-1} V_{i,t} \left(\frac{W_t}{P_{i,t}^V} \right)^{-\sigma_i^V}$

Industry Production and Trade Technology Functions

Value added output

$$V_{i,t} = A_{i,t}\left[\theta_i^{N\frac{1}{\sigma_i^V}}\left(\lambda_{i,t}^N NH_{i,t}\right)^{\frac{\sigma_i^V-1}{\sigma_i^V}} + \theta_i^{K\frac{1}{\sigma_i^V}} K_{i,t-1}^{\frac{\sigma_i^V-1}{\sigma_i^V}}\right]^{\frac{\sigma_i^V}{\sigma_i^V-1}}$$

Domestic output

$$D_{i,t} = \min_{\{V_{i,t},J_{i,t}\}}\left(\left(1-\sum_j \beta_j\right)V_{i,t}, \sum_j \beta_j J_{ji,t}\right)$$

Total supply (production)

$$Y_{i,t} = \left[\alpha_i^{\frac{1}{\sigma_i^Y}}\left(D_{i,t}\right)^{\frac{\sigma_i^Y-1}{\sigma_i^Y}} + (1-\alpha_i)^{\frac{1}{\sigma_i^Y}}\left(M_{i,t}\right)^{\frac{\sigma_i^Y-1}{\sigma_i^Y}}\right]^{\frac{\sigma_i^Y}{\sigma_i^Y-1}}$$

Total use (distribution)

$$Y_{i,t} = \left[\phi_i^{\frac{1}{\sigma_i^T}}\left(X_{i,t}\right)^{\frac{\sigma_i^T-1}{\sigma_i^T}} + (1-\phi_i)^{\frac{1}{\sigma_i^T}}\left(E_{i,t}\right)^{\frac{\sigma_i^T-1}{\sigma_i^T}}\right]^{\frac{\sigma_i^T}{\sigma_i^T-1}}$$

Notation

Where V is value added output; D is domestic output, Y is total supply/use; J is intermediate inputs; M is imports, X is exports, E is domestic demand; NH is total hours worked; K is capital; A *is* total factor productivity; λ^N is labour augmenting technical change; ξ is capacity utilisation; θ, β, α and ϕ represent input shares; and, σ is elasticity of substitution/transformation.