Modelling Fiscal Policy with OLGA

Treasury’s OverLapping Generations model of the Australian economy

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# Abstract

To meet the needs of a broad set of internal and external stakeholders now and into the future the Australian Treasury maintains a significant macroeconomic modelling capability. Treasury’s current in-house capability is similar to that of the US Congressional Budget Office (CBO) and the Joint Committee on Taxation (JCT). These agencies have a suite of macroeconomic models to quantify the general equilibrium effects of policy on economic activity, household welfare and public finances. In this paper we introduce one of the models in that suite, Treasury’s overlapping generations model of the Australian economy (hereafter OLGA). OLGA has been developed by economists in Treasury’s Macroeconomic Analysis and Policy Division to support Treasury’s counterfactual fiscal policy analysis. OLGA is a small open economy variant of the well-known lifecycle model developed by Auerbach and Kotlikoff (1987). It has been calibrated to Australian data and policy settings.

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# Introduction

This paper introduces Treasury’s overlapping generations model of the Australian economy (hereafter OLGA). OLGA has been developed in-house by economists in Treasury’s Macroeconomic Analysis and Policy Division to support Treasury’s counterfactual fiscal policy analysis. OLGA is a small open economy variant of the well-known lifecycle model developed by Auerbach and Kotlikoff (1987). It has been calibrated to Australian data and policy settings.

While no single economic framework can capture all the relevant dimensions of fiscal policy, overlapping generations (OLG) models have become one of the key tools in the economics profession for quantifying the effects of fiscal policy on macroeconomic aggregates, household welfare and public finances.[[3]](#footnote-4) This reflects a number of desirable features of modern OLG models recognised by leading academics and fiscal agencies including the US Congressional Budget Office (CBO) and the Joint Committee on Taxation (JCT):

* OLG models are general equilibrium, meaning that they take full account of second round (and subsequent) effects in addition to the direct effects of policy changes. This includes taking account of the feedback effects of price changes and the spillovers from one market to another market.
* Households and firms are rational, forward-looking and maximise explicit objective functions subject to their budget constraints. This means agents’ expectations are consistent with the economic framework and the modelled outcomes which permits an explicit comparison of household welfare across alternative fiscal policies.
* While the economy exists in perpetuity, households have a finite lifespan with the population comprising many overlapping generations of different ages at any point in time. This permits the inter‑generational effect of fiscal policies to be identified.
* The inclusion of households of different skill levels, as well as different ages, introduces household heterogeneity. This allows an assessment of the effect of different policies on the distribution of household income and wealth, and on household welfare. OLG models are therefore able to quantify the effect of fiscal policy on cross-section and lifetime inequality.
* OLG models include a separate fiscal authority (hereafter government) that has a full set of taxation instruments at its disposal, undertakes expenditure and transfers, and is subject to a budget constraint that is binding in the long run. This means that the government can debt‑finance a budget deficit in the short run but necessarily rules out policies that would result in unsustainable public finances in the long run. Changes in policy that have permanent fiscal effects must be offset by another policy change to ensure long-run budget neutrality. In contrast to government, households have a finite horizon. An important implication of these different decision horizons is that strict Ricardian equivalence does not hold which means the model can identify fiscal policy that improves household welfare.

These features set OLG models apart from other macroeconomic modelling options available in Australia such as:

* Time series econometric models, macro-econometric models and other ‘reduced form’ approaches (for example, so called recursive CGE models) which lack a well-defined welfare criterion and sufficient detail to quantify the activity and welfare effects of alternative fiscal policies.
* Static general equilibrium models, including those previously used by Treasury and other researchers to study effects of different taxes in Australia (see for example, Cao et al. (2015), Murphy (2016) and KPMG (2016)), which ignore potentially important transitional dynamics and costs associated with adjusting aggregate capital or relocating capital across sectors.
* Representative agent dynamic general equilibrium models which are unable to consider intra- and inter-generational dimensions of fiscal policy because they assume a single infinitely-lived household.

Presented in this paper is OLGA Version 1.0 which has several key elements worth noting. First, the model has 75 generations of forward-looking, finite-lived households with five different skill types, ranging from high to low. This captures rich heterogeneity amongst Australian households. Second, the model has a number of production sectors based on the input-output table. This captures critical supply and demand relationships between producers and consumers within the domestic economy, and Australia’s trade and financial linkages with the rest of the world (ROW). Third, the model has a detailed representation of major Australian taxes, age pension and other transfers and aggregate government spending. This allows Treasury to model Australian policy at the level of detail required for decision making. Finally, as a general equilibrium model including the indirect effects of a policy proposal, OLGA provides a comprehensive assessment of the so called ‘dynamic cost’ (or second round effects on the cost) of a policy proposal.

OLGA is a deterministic model which means it does not account for business cycle fluctuations and so is not well suited to modelling stabilisation policy. As such, it is best suited to analysing the allocative and distributional effects of fiscal policy.

Treasury follows best practice employed by other fiscal agencies (for example, CBO or JCT) by relying on a suite of economy-wide models to produce forecasts and policy advice. In addition to OLGA, Treasury’s suite includes:

* Treasury’s Macroeconometric Model of Australia (EMMA) (see Bullen, et al. (2021)). EMMA is central to Treasury’s forecasting and policy advising process. EMMA is a large-scale macroeconometric model of the Australian economy. It captures the rich interaction between aggregate demand (which includes consumer spending, business investment, residential investment, government spending, and net exports) and aggregate supply (which includes the endogenous and exogenous factors that determine Treasury’s estimate of potential output) in the economy. The interaction of aggregate demand and aggregate supply determines the forecasts of the other variables in the model, including inflation, interest rates, the unemployment rate and incomes. EMMA models policy using effective rates (for example, the average person income tax rate) or lump sum measures (for example, aggregate transfers to households).
* Treasury Industry Model (TIM) (see Carlton et al. (2023)). TIM has been developed to be Treasury’s principal tool for industry policy analysis. It is a dynamic multisector general equilibrium model of the Australian economy. At its core TIM is a small open economy version of the neoclassical growth model known as the Ramsey Cass Koopmans model. In contrast to typical neoclassical growth models, TIM has considerable firm heterogeneity in the form of 114 forward-looking, infinitely-lived firms that represent Australian industries. TIM includes significant detail on the linkages between industries and is well suited to studying the effects of industry-specific policies.
* Detailed analysis of the economic effects of policy proposals is generally conducted using OLGA, TIM or detailed bottom up analysis informed by empirical estimates from the literature. For materially large policies the effects are added to the EMMA baseline directly or by adjusting effective policy variables to yield a policy-consistent forecast of economic activity.

The development of OLGA, and Treasury’s macroeconomic model development program more generally, has been supported by academic advisers with deep expertise in general equilibrium modelling, including OLG models.

The remainder of the paper is organised as follows: Section 2 describes in detail the theoretical structure of OLGA; Section 3 describes the techniques used to solve OLGA; Section 4 details the calibration methodology, including data sources; Section 5 describes the welfare criterion; and the paper concludes in Section 6. Other technical detail including OLGA’s first order conditions can be found in Appendices A to G.

# The model

Overview

OLGA is a variant of the well-known lifecycle model with overlapping generations developed by Auerbach and Kotlikoff (1987). The primary difference is that OLGA is a small open economy model calibrated to Australian data. The model includes households of multiple generations that are composed of workers with different skill levels, and a number of production sectors with different supply and demand side characteristics. A schematic diagram of OLGA is provided in Appendix A.

There are 75 overlapping generations of households in OLGA. Each generation enters the model at age 21 with a known earning ability, which reflects their age and level of skill. Households are assumed to be uniformly distributed across five skill types. Households save during their working lives and rely on savings during their retirement. Households’ savings are managed by a notional funds manager. High net worth households leave intended bequests. Some households are eligible for means‑tested retirement benefits at age 66. Households do not live past age 95.

In contrast to most OLG models, OLGA has seven production sectors, where each sector is characterised by a single representative firm that maximises the market value of the firm on behalf of its shareholders. Firms employ labour, capital, and intermediate inputs to produce final and intermediate goods and services for domestic use or export. Domestically produced goods compete with differentiated goods and services supplied by foreign producers. Firms manage capital investment decisions, and source funds from households and foreign investors via a notional funds manager.

OLGA incorporates a detailed representation of Australia’s fiscal policies. These include major taxes such as progressive personal income tax with a range of tax offsets, corporate tax, goods and services tax, and major government transfers such as the means-tested age pension and other age-dependent payments. The government also has a fixed bundle of spending. The tax revenue finances government transfers and spending. The government relies on funds sourced from domestic and foreign investors to meet temporary primary deficits.

### Households

#### Demographic structure

##### Population growth

The model is populated by households spanning 75 generations. Following the broader literature, a new household enters the model at age 21 and lives to a maximum age of 95. The age of a household is denoted by, while the number of households of ageat the beginning of year  is denoted by .

Given the model’s time interval of one year, the total population at the beginning of year is  which is the sum of the model’s 75 generations:



Population growth is exogenous to the model with the size of an incoming generation assumed to grow over time as follows:



whereis the growth rate at time of the population of 21‑year‑old households.

Households of age survive until next period with a conditional probability of and a cumulative probability  of surviving until age . This implies that the size of a generation declines over time. For example, the size of a generation that enters the model at the beginning of yearwill bewhen the cohort reaches age:



The implied growth of the total population is given by:



whereis the growth rate at time  of the total population.

##### Skill and technical progress

A worker’s earning ability is an exogenous function of their age, skill level and the level of labour augmenting technical progress. Each generation is assumed to consist of households with five different skill levels denoted by. The size of the population of ageand skill at time is denoted by, with each skill type accounting for one-fifth of the generation (that is, ). A worker’s earning ability is assumed to be the product of their relative labour efficiencyand labour augmenting technical progress:



Relative labour efficiency is a time-invariant function. To match lifetime participation and earnings profiles relative labour efficiency is assumed to be hump shaped, reflecting the accumulation of human capital over the course of the household’s early life and following peak ability around age 40, a decline in ability that eventually settles at zero.

Labour-augmenting technical progressis an exogenous process (determined outside the model) which is assumed to grow at rate, such that:



Following the literature in describing the steady state of the model we normalise growing variables by their underlying exogenous trend. Normalised variables are denoted by a lower case. First, non-labour household variables and real wages are normalised by the level of labour augmenting technical progress. For example, the normalised consumption of a household of age and skill  at time is. Second, aggregate labour variables are normalised by aggregate population. For example, the normalised population of ageand skillat time is. Finally, aggregate non-labour variables are normalised by the product of labour augmenting technical progress and the aggregate population, which has a growth rate equal to. For example, the normalised aggregate exports at time is.

#### Preferences

Households are assumed to maximise their lifetime utility. A representative household of skill type, who enters the model at the beginning of year, is assumed to have the following lifetime utility function:



where:  is the household’s discount factor; ,, andare the household’s consumption, leisure, and beginning-of-period savings at year.

Given that there is a simple linear relationship between the year a household enters the model, their age and time, going forward the paper will denote household variables using only a time subscript and an age superscript.

##### Instantaneous utility

Instantaneous utility is assumed to be a constant relative risk aversion (CRRA) function of the household’s aggregate consumption and leisure:



Where  is the inverse of the intertemporal elasticity of substitution andmeasures the household’s preference for consumption over leisure.

Consumption is a composite good sourced from a notional distributional sector which is explained in further details below.

##### Bequests

Following De Nardi (2004), we assume ‘warm glow’ altruism with the household deriving utility from bequeathing their residual savings following their death at time  and age  accordingly:



The term measures the strength of the household’s bequest motive, whilemeasures the extent to which bequests are a luxury good. A bequest motive will result in the build-up of savings that is more than the household would otherwise accumulate over its lifetime. Forthe marginal utility of small bequests is bounded which suggests bequests are a luxury good.

For simplicity, we assume that savings of households who die at time  are distributed equally to their heirs who are assumed to be the surviving population of the same skill type:



#### Labour supply

For ease of exposition and without loss of generality, households are assumed to have one unit of time available for market-based work or leisure:



Aggregate labour supply measured in units of time is given by:

,

while aggregate labour supply measured in relative efficiency units is given by:



Efficiency units are perfectly substitutable. As such, workers earn a common before tax wage ofper efficiency unit. The hourly wage received by a household of ageand skill,is the product of the common wage rate per efficiency unitand the household’s relative labour efficiency:



#### Savings

Households have a single savings instrument which is assumed to be managed by a notional financial manager. The value of household savings at the beginning of time  is denoted by . The underlying asset is a portfolio of domestic and internationally traded assets. The before personal income tax rate of return for this asset is. Details of the funds market are discussed in a later section.

Households enter the model with zero savings net of bequests (that is, ). However, they can borrow at the before personal income tax rate of returnsubject to the following borrowing constraint:



where is the credit limit for households to borrow. For the current version of OLGA, we have for all  andfor all . Intuitively, this allows households under the age of 60 (but not over) to borrow against their future income. This means that households who are eligible for age pensions are not able to borrow against future age pension payments.

#### Personal income tax

Households are subject to personal income tax (PIT). PIT is levied on the following tax base:



where: is the ratio of income tax deductions and exemptions to gross income; is the means-tested age pension; is other taxable transfers from the government such as unemployment benefits and the disability pension; and taxable labour income before deductions is defined as follows:



The capital income tax base for the purpose of calculating personal income tax payableis:



where:is the taxable return on savings at time , which reflects the actual distribution/flow of earnings via interest, dividends and off-market share buy-backs; and denotes changes to the value of savings due to changes in asset prices. Taxable returns are also affected by discounting and credits.

Personal income tax payable (before franking credit refunds and any offsets) varies according to the following progressive schedule with *J* brackets:



where:  is the closest threshold that is larger than; is the upper threshold for income bracket; and income in bracket to is subject to marginal tax rate.

The gross personal income tax liability can be reduced by a number of income-dependent tax offsets:



where:  is the maximum amount for offset ; and  are income-test thresholds; and  is the taper rate, which indicates the amount the offset is reduced by for each additional unit of income above .

The net personal income tax liability is equal to the gross liability less franking credits and tax offsets. Tax offsets are non-refundable and are therefore applied before franking credits, resulting in a net personal income tax liability as follows:



where:  is the franking credit received by the household.

Given the personal income tax system described above, the effective marginal personal income tax rate will be a combination of a given household’s marginal tax rate , and the taper rates  for the tax offsets that the household receives.

#### Transfers

##### Age pension

A household may be entitled to age pensionafter reaching the pension eligibility age, subject to asset and income means testing. Households receive the smaller of the pensions implied by the asset and income tests:



where:  is the asset-tested pension;  is the income-tested pension; and  is the pension eligibility age.

The asset-tested pension is determined according to the following rule:



where:  are assessable assets (, where are exempt assets);  and are the asset-test thresholds; and  is the taper rate, which indicates the amount the pension is reduced by for each additional unit of asset above .

The maximum pension payment  is in turn given by:



Where and  are the levels of household average labour supply and the replacement ratio consistent with the current pension rule. This mimics the reality that age pensions are indexed to current price levels and benchmark earnings.[[4]](#footnote-5)

Similarly, the income-tested pension is determined according to the following rule:



where is the sum of a household’s labour income and deemed income from savings:



where:and are income-test thresholds; and  is the taper rate which indicates the amount the pension is reduced for each additional unit of income above .

The deemed income from savings is derived using deeming rates and a deeming threshold :



##### Other transfers

Households also receive lump-sum transfers (if positive) or pay lump-sum tax (if negative) denoted by. The lump-sum transfer is the sum of government benefits and transfers that are specific to certain households and a general lump-sum transfer/tax that applies to all households.

#### Household budget constraint and optimisation problem

The household’s flow budget constraint is as follows:



Hereis the after-tax price of household’s aggregate consumption bundle at time .

The household’s objective is to maximise lifetime utility (1), subject to the budget constraint (7). Further details of the households’ optimisation problem are provided in Appendix B.

### Production sector

For simplicity, we assume that there are  sectors with a single firm in each representing a large number of individual firms with homogenous technology trading in a perfectly competitive market. In other words, there is one price-taking firm in each sector.

#### Production technology

The firm’s production technology is represented by a constant elasticity of substitution (CES) function:



where for sector: is total factor productivity;  is the sector’s use of labour input measured in relative efficiency units;[[5]](#footnote-6)is the sector specific capital stock;  is a composite intermediate input; , , and  are the CES weights for each input, with ; and  is the elasticity of substitution for factors of production.

The normalised version of equation is:



##### **Labour input**

Labour is supplied by households. Efficiency units of labour are perfectly substitutable across sectors, so in equilibrium workers, in different sectors, receive a single wage. As such firms and workers are indifferent to the sectoral allocation of workers by efficiency unit:



##### **Capital input**

Firms own and manage their variable productive capital. The capital stock of a representative firm in sector evolves according to the following accumulation identity:



Investment is a composite good sourced from a notional distribution sector which is explained in further details below.

Following Lucas (1967), we assume that the firm faces capital adjustment costs. In particular, we assume that the firm faces quadratic adjustment costs such that there is an increasing loss of output as the investment to capital ratio moves further away from its steady state level:



where governs the size of the adjustment cost.

##### **Intermediate inputs**

Production requires the consumption of intermediate inputs in the period in which they are produced. For example, the production of bread requires intermediate inputs such as flour, yeast and salt.

While intermediate inputs are directly sourced by firms from each production sector, for ease of exposition we assume that firms purchase a sector-specific composite of intermediate goods from a notional distribution sector which is described in more detail below.

##### **Gross output**

Finally, the firm’s gross output is summarized by the following:



#### Corporate finance

The representative firm owns the capital stock and makes production and investment decisions in order to maximise the market value of the firm. In addition to equity the firm's capital is also financed by borrowing in the corporate debt market. For simplicity we assume that firms maintain fixed debt to equity ratios. Furthermore, to capture features of the Australian tax system equity is divided into equity which pays dividends or capital gains.

The value of the firm’s capital financed by corporate debt is:



where: is the market value of a unit of capital;  is the capital stock at the beginning of period ; and  is the share of the firm’s capital financed by corporate debt. For Version 1.0 of OLGA,  is exogenous.

Corporate debt is assumed to be an internationally traded one period bond which has a required rate of return of, which means that each period, in addition to the principal, the firm must also pay to bond holders.

The representative firm’s equity (that is, the market value of the firm) is:



Corporate equity is also traded internationally. We assume that the required rate of return for equity is set by the global funds market based on financial assets with the same risk characteristics.

We assume that a notional funds manager raises funds on behalf of firms. The funds manager sources funds from domestic households and foreign investors. Details of the funds market are discussed in a later section.

#### Market value of the firm

The no-arbitrage condition requires that:



where:  is the net cash-flow from the firm to shareholders;  is the market value of the firm’s equity at the beginning of period ; andis the required rate of return for equity. This condition implies that investors are indifferent to holding equity (with earnings returned via dividends or capital gain) and holding other financial assets with the same risk characteristics. The net cash-flow may reflect outflows due to dividends and/or off‑market share buy-back and inflows from the issuance of new equity.

Using normalised variables, we have:



where:  is the net cash-flow from firms to households; andis the market value of the firm’s equity at the beginning of period .

This implies:



Forward substitution of the no-arbitrage condition (assuming there are no self-fulfilling speculative asset-price bubbles) implies the market value of the firm’s equity is equivalent to the present value of current and all future net cash-flow to shareholders:



#### Corporate income tax

The firm’s earnings before interest, tax and amortisation is:



where:  is the producer’s price;  is real gross production;  is the payroll tax rate;  is the sector’s use of labour inputs measured in efficiency units; is a composite of intermediate inputs from all production sectors (including itself); and is the price of the composite intermediate good.

Corporate income tax (CIT) is payable to the government. Given deductions on interest payable on corporate debt, investment allowances and depreciation, a firm pays the following level of CIT:



where:  is the effective CIT rate;  is the proportion of economic depreciation that is tax deductible;  is the economic depreciation rate;  is an additional investment allowance that can be deducted as an expense under the tax system;  is the level of investment and is market price of investment goods.

The effective CIT rate is the product of the statutory CIT rateand the CIT taxable share of the sector:



where  reflects the share of firms (on an output value basis) that are incorporated and therefore subject to corporate tax. The current version of the model assumes  is exogenous which means firms cannot optimise their value subject to legal structure. The main implications of this assumption are that changes to personal income tax settings have no direct effect on the investment decisions in the unincorporated sector, and that depreciation of capital in the unincorporated sector cannot be deducted as an expense for tax purposes. Similarly, there is currently no provision for carrying forward losses in the model. These limitations will be addressed in future model developments.

#### Payroll tax

Firms are also subject to payroll tax, which is payable to the government:



#### Firm’s budget constraint

The firm is subject to the following flow budget constraint:



Earnings plus borrowing must equal the sum of corporate bond repayment (principal and interest), corporate tax, investment and net cash flow paid to shareholders (dividends or off-market share buy-backs less new share issuance).

#### Dividend policy

For simplicity, we assume the firm retains sufficient earnings to cover the cost of replacing depreciated capital . After interest  and tax , the balance is returned as either a dividend or off-market share buy-back collectively denoted by :



This implies the following rule for the new issuance of equity:



The new issuance of shares is equal to the cost of investment in excess of depreciation less the change in the value of the firm’s capital that is financed by bonds. At the steady state, this rule implies new issuance is equal to the growth of the capital stock financed by equity:



#### Firm’s optimisation problem

The representative firm’s objective is to maximise the market value of the firm by choosing the level of production (that is, the level of capital, labour and intermediate inputs) subject to the budget constraint , production technology , and the market prices of output, investment, labour and intermediate goods. Further details of the firm’s optimisation problem are provided in Appendix C.

### Rest of the world

#### Global goods and services market

For imports of goods and services, we assume the foreign sector supplies  at the exogenously determined price. This implies that domestic demand cannot affect the price. In other words, the supply of imports is perfectly elastic at this price. Imports compete with differentiated goods of the same type produced domestically. The extent to which they compete is dictated by the degree of substitutability. For ease of exposition, we assume there is a notional distribution sector which aggregates domestic and imported varieties of goods. This approach is explained in detail in the next section.

For exports of goods and services, we adopt a richer specification than typically assumed in small open economy models by recognising that export demand is affected by substitutability for varieties of the same good; substitutability for other goods; and the market power of Australian producers. In a similar way to global economic models (for example, Hertel (1997); McKibbin and Wilcoxen (1998)), we employ a representative consumer in the ROW who solves a nested utility maximisation problem. At the top of the nest is the choice between different types of goods. Following this is the choice between varieties of the same good that are produced in different exporting countries.

As derived in Appendix D, the foreign consumer’s optimal level of demand for Australian exports are:



where for good:is the level of Australian exports;is the level of foreign aggregate demand;  is the CES weight for the specific good in foreign aggregate demand;  is the CES weight for Australian exports in foreign demand for the specific good;  is the elasticity of substitution between different types of goods; and is the elasticity of substitution between varieties of the same good from different exporters. Furthermore, is the price index of the foreign consumer’s aggregate demand, andis the price index of goodfor the foreign consumer that is a CES aggregate of the prices of exports the from Australiaand the ROW:



Bothand are assumed to be exogenous.

The export demand function for goodmakes it clear that Australian exports depend on the substitutability of Australian and ROW varieties of good, , the substitutability of all goods and the market power of Australian producers of good ,. When competing with other varieties, higher substitutability implies export demand is more sensitive to price changes, while greater market power implies export demand is less sensitive to price changes. When competing with other goods, higher substitutability implies export demand is more sensitive to aggregate price changes, while greater market power implies greater influence of Australian prices on aggregate prices.

#### Global funds market

Foreign capital is assumed to be supplied as one period bonds or equity. Irrespective of the end use, the supply of capital is assumed to be perfectly elastic at the global required after tax rate of return for that investment type. Required rates of return vary across investment types due to different risk premia. We assume a benchmark rate equal to the global required after tax rate of return for government bonds denoted. Following the empirical literature, equity is assumed to have a larger risk premium than corporate debt, which in turn has a larger premium than sovereign debt.

Finally, Australian investors can also undertake portfolio investment offshore at an assumed foreign after-tax rate of return.

#### National budget constraint

Given the quantities and prices of imports and exports, as well as the global required rates of return, the national budget constraint is as follows:



where: are household assets held abroad; are gross assets held in Australia by foreigners; andis the average after-tax rate of return for the foreigner’s investment portfolio.

### Distribution sector

As discussed above, for ease of exposition we assume that there is a notional domestic distribution sector. The distribution firm combines imported goods and domestically produced goods to form composite consumption , investment , intermediate inputs , and government spending which are sold to households, firms and the government. Further details of the distribution sector’s optimisation problem are provided in Appendix E.

#### Consumption goods and services

Household consumption  is a nested CES aggregate of individual consumption goods which reflects household preferences. At the top tier, the distribution sector notionally allocates total expenditure to a basket of goods , which are in turn composites of domestic and imported varieties. The distribution sector picks the consumption bundle that maximises household utility subject to the allocated budget:



subject to:



where:  is the elasticity of substitution;  is the CES weight for different goods with; and  is the market price including all taxes and subsidies on each composite good.

At the bottom tier, the distribution sector notionally allocates expenditure to a basket of goods composed of domestically produced and imported  varieties. The distribution sector picks the consumption bundle that maximises household utility subject to the allocated budget:



subject to:



where: is the elasticity of substitution;is the CES weight for imported goods; is the statutory goods and services tax (GST);  is the share of goods subject to GST; andis an effective ad valorem tax rate that captures the net effect of duties, subsidies and other taxes associated with the supply of consumption goods and services.

#### Investment goods and services

Capital formation of sectoris assumed to be a composite investment good  which is an aggregate of different types of goods  which in turn is a composite of domestically produced and imported varieties.

At the top-tier, the distribution sector’s objective is to maximise the value of the firm’s gross fixed capital expenditure by combining different types of investment goods subject to the firm’s investment preferences, captured by a CES function, and the firm’s allocated budget:



subject to the budget constraint:



where: is the elasticity of substitution;  is the CES weight for variety  with ; and is the market price of each composite good for investment of the sector.

At the bottom-tier, the distribution sector’s objective is to maximise the firm’s value of gross fixed capital expenditure of variety  by combining domestic and imported varieties of investment goods subject to the firm’s investment preferences, captured by a CES function, and the firm’s allocated budget:



subject to:



where:  is the elasticity of substitution;  is the CES weight for imported goods;  is share of investment goods subject to the GST; and is an effective ad valorem tax rate that captures the net effect of duties, subsidies and other taxes associated with the supply of investment goods and services.

#### Intermediate goods and services

In a similar vein, the distribution sector produces aggregate intermediate goodby combining different inputs , which are in turn composites of domestically producedand importedvarieties.

At the top-tier, the distribution sector’s objective is to maximise the value of the firm’s intermediate input expenditure by combining different types of intermediate goods subject to the firm’s production requirements, captured by a CES function, and the firm’s allocated budget :



subject to the expenditure constraint:



where:  is the elasticity of substitution;  is the CES weight for variety  with; and is the market price of intermediate good .

At the bottom-tier, the distribution sector’s objective is to maximise the firms value of intermediate goods expenditure on variety  by combining domestic and imported varieties of intermediate goods subject to the firm’s production preferences, captured by a CES function, and the firm’s allocated budget:



subject to:



where: is the elasticity of substitution; is the CES weight for imported goods;  is share of good k that is subject to the GST; and  is an effective ad valorem tax rate that captures the net effect of duties, subsidies and other taxes associated with the supply of intermediate goods and services.

#### Government spending

Government spending also has a two-tiered preference structure, although it has a different allocation mechanism for goods and varieties.

At the top-tier, the government is assumed to undertake spending with a fixed bundle of goods and services supplied by the distribution sector. Given the market price of each good, total government spending is:



Here  is the price of composite good purchased by the government, and real government expenditureis assumed to be a CES aggregate of the bundle:



With  denotes the elasticity of substitution, and  denotes the CES weight for variety with .

At the bottom-tier, the distribution sector optimally allocates fundson behalf of the government to domestically producedand importedvarieties of good j:



subject to:



where: is the elasticity of substitution; andis the CES weight for imported goods.

#### Goods and services tax

The Australian GST framework is summarised in Table 1, based on information from the Australian Taxation Office (ATO, 2022). There are three possible outcomes: the sector is ‘subject to GST’ if the goods and services sold by the sector is taxed at the GST rate and the sector is credited for the GST paid on its inputs; the sector is ‘GST free’ if the goods and services sold by the sector are not subject to GST and the sector is credited for the GST paid on its inputs; and the sector is ‘input-taxed’ if the goods and services sold by the sector are not subject to GST and the sector is not credited for the GST paid on its inputs.

We capture this variation through coverage factors that are specific to the type of good and service and its stage of production. For example, if a sector falls under the ‘subject to GST’ category all GST paid on intermediate inputs and investment goods will be refunded by the government:

,

while the full statutory rate will apply to goods sold to the consumer:

.

However, the sectoral detail in the model is typically not at the level of individual goods and services where the coverage factors are zero or one, so we must use value-weighted shares of individual consumption, investment or intermediate goods to estimate the coverage factors that applies to the sectors in the model. These estimated coverage factors lie between zero and one.

Table 1: Taxation under different GST treatments

|  |  |  |
| --- | --- | --- |
| **Treatment** | **Output subject to GST** | **Credits for GST on inputs** |
| **Subject to GST** | Yes | Yes |
| **GST-free** | No | Yes |
| **Input-taxed** | No | No |

Source: ATO (2022).

Given the statutory GST rate and the coverage factors, the total GST collected from the distribution sector is:



#### Other indirect taxes

Given the effective ad-valorem tax rates for other indirect taxes, the revenue collected by the government is:



### Government

The government is defined as the ‘general government sector’. For Version 1.0 of OLGA, detailed modelling of the government sector has been confined to Commonwealth government taxes and transfers, with rules of thumb for state and local taxes and government spending. Detailed modelling of state and local taxes including land tax and municipal rates will be incorporated in future model development. Similarly, detailed modelling of general government spending will be considered in a development module dedicated to government spending.

#### Revenue

The government raises revenue through various forms of taxation, with the total tax collected denoted. These taxes include personal income tax, corporate income tax , payroll tax , goods and services tax, other ad valorem taxes , and withholding taxes which are the sum of withholding tax on income earned by non-residents from investment in Australian government bonds and corporate bonds:





#### Payments

##### **Spending**

As noted above, the government is assumed to undertake spending on a fixed bundle of goods and services supplied by the distribution sector. This bundle gives the aggregate real government expenditure  at price  as per equations (26) and (27).

Following the broader literature, government spending does not enter the household utility function. Both the level of government spending and the utility it provides will be the subject of the government spending development module.

##### **Transfers**

Households can access the age pension (subject to means-testing) and other social transfers (see age pension and social transfer functions above). The total cost of the age pension is, while the total cost of other transfers is :





#### Government budget constraint

The government’s flow budget constraint is:



Where is the level of government net debt at the beginning of period .

In a model where households have perfect foresight, the government cannot incur unbounded liabilities or accumulate unbounded assets in the long run. To rule out this possibility we assume that the government credibly commits to a fiscal rule that ensures its debt to GDP ratio is stabilised at a given target within a finite period. In other words, the government must adopt a funding offset that yields a path of primary budget balances sufficient to achieve and stabilise debt at its target level by the specified date. This means that permanent increases in spending or reductions in tax revenue must be matched by permanent increases in other taxes or reductions in spending. Similarly, temporary increases to spending or reductions in tax revenue must be either financed by a temporary offset sufficient to stabilise debt at its original level, or a permanent increase in taxation or cuts to spending sufficient to fund the higher interest expenditure that comes with a higher debt level.

### Financial sector

The financial sector allocates household savings to domestic and foreign assets. Under the small open economy and perfect capital market assumptions any shortfall in funds is sourced from the foreign sector, with the foreign investor being the marginal investor in all asset markets.

#### Household savings

Domestic households supply the following level of funds:



Investment decisions are managed by a notional funds manager.

The allocation of household savings follows a nested structure. At the top tier, households are required to allocate a share of savings to ownership of dwellings with the remainder allocated to non‑dwelling assets as described in detail below.

OLGA Version 1.0 assumes the Australian dwellings sector is fully owned by Australian households. This means the savings allocated to dwellings must equal the market value of the dwellings sector:



Where  is the share of household savings allocated to dwellings. In this version of OLGA, houses are assumed to be owner-occupied so the income stream from housing is not taxed and there is no need to distinguish between equity holdings that return earnings via capital gain or dividends. This limitation will be addressed in a development module dedicated to the taxation of housing.

At the second tier, the remaining share of household savings  is allocated to non-dwelling assets. These savings are distributed across a fixed-weight portfolio of domestic and foreign assets. The domestic component includes: domestic corporate and government bonds, with weights of and  respectively; and equity in domestic firms in all non-dwelling sectors with a weight of  for firms that distribute earnings as fully franked dividends, and a weight of  for firms that distribute earnings as capital gains. The foreign component is assumed to be a composite of foreign assets (foreign bonds and equity) with weight equal to.

For Version 1.0 of OLGA we assume that the portfolio for household savings allocated to non‑dwelling assets is fixed. This assumption will be relaxed in a development module dedicated to the taxation of savings.

#### Domestic government bonds

For simplicity we assume the government issues one period bonds. The assumed global required after tax rate of return for sovereign/government debt is . Foreign investors receiving income from bonds must pay withholding tax at the rate . A country risk premium implies the following global required before tax rate of return on Australian government bonds:



Domestic government bond holders who are subject to a marginal personal income tax ratewill earn the following after tax return:



Based on the fixed asset allocation shares described above, the value of government bonds held domestically is:



This implies the following foreign holding of Australian government bonds:



and withholding tax revenue:



#### Domestic corporate bonds

For simplicity we assume firms issue one period bonds. The global required return on corporate bonds is as follows:



where:  is the risk premium on corporate bonds; and  is the withholding tax rate.

Domestic corporate bond holders who are subject to a marginal personal income tax rate  will earn the following after tax return:



Given the fixed asset allocation of shares described above the value of corporate bonds held domestically is:



This implies the following foreign holding of Australian corporate bonds:



and withholding tax revenue:



#### Domestic equity

The required rate of return for a foreign equity investor will depend on the way earnings are distributed. Foreign investors do not pay tax in Australia on capital gains earned on Australian assets which implies the following required rate of return to foreign equity holders when their return is realised as a capital gain:



whereis an equity premium.

In contrast, foreign investors must pay withholding tax on dividends, which implies the following required rate of return to foreign equity holders when their return is realised as a dividend:



Given , the lowest cost foreign investor (aka the marginal foreign investor) is one that realises their return as a capital gain. This implies that the global required rate of return on domestic equity is:



In this version of the model we do model the optimal allocation of funds between non-dwelling assets by assuming that households invest in firms that pay dividends as well as firms that return earnings as a capital gain according to an exogenous fixed split  and . Domestic and foreign equity investors earn the same rate of return. It follows that a domestic investor, subject to marginal personal income tax rate , who realises their return as a dividend will have the following after tax return:



while those that realise their return as a capital gain will have the following after-tax return:



where  is the capital gains discount.

Based on the asset allocation shares described above, the equity in Australian firms held domestically (including in the dwellings sector) is:



This implies the following foreign holding of equity in Australian firms:



#### Household’s foreign investment

Finally, domestic investors are assumed to earn a before Australian tax rate of return of  on the composite of foreign assets (bonds and equity):



Where  is the risk premium of the foreign portfolio over the sovereign borrowing rate.

Based on the asset allocation shares described above, foreign assets held domestically is:



#### Household’s return on savings

The household’s implied before personal income tax rate of return on savings is as follows:



Because ownership of dwellings is exempted in Version 1.0 of OLGA, the rate of return on household savings for the purpose of calculating personal income tax payable is as follows:



This reflects that dividends are grossed up to account for franking credits, while earnings returned via share buyback reflect a discounted capital gain. Returns from corporate bonds, government bondsand foreign assets are taxed at full value. As we do not model superannuation separately in this version of OLGA, there is no concessional treatment of savings through superannuation.

For simplicity households are assumed to turn over their savings every period. As such, they are subject to tax/credit on the capital gain/loss resulting from the change in asset prices in that period. Foreign investors are not subject to capital gains tax. Corporate bonds, government bonds and the household’s foreign portfolio do not experience capital gain or loss because they are assumed to be one period securities. This means equity is the only household asset subject to price change.

#### Total foreign liabilities

The total foreign liabilities of the Australian economy and the related portfolio weights are then:



where:is the implied share of foreign liabilities accounted for by foreign holdings of Australian government bonds;is the implied share of foreign liabilities accounted for by foreign holdings of Australian corporate bonds; and is the implied share of foreign liabilities accounted for by foreign holdings of equity in Australian firms.

The implied after-tax rate of return on this portfolio is:



### Competitive Equilibrium

Given the demographic structure, relative labour efficiency, labour augmenting technical progress , government policy (spending, transfers and tax), foreign prices, the global required after tax rate of return for sovereign/government debt, risk premia and the initial distribution of household asset holdings, a competitive equilibrium corresponds to a sequence of pricesand a corresponding sequence of decisions by householdsand firmsthat satisfy the following conditions:

1. Households maximise intertemporal utility subject to their budget constraint ;
2. Firms maximise the market value of the firm subject to their budget constraint ;
3. The government’s budget constraint is satisfied;
4. Labour and capital markets clear;
5. Goods markets clear:



If the demographics and government policies are stabilised in the long term, then a balanced-growth path exists where, with the exception of real wages which grow at constant rate, relative prices are stationary and all other variables (in per capita terms) are growing at the constant rate of.

Solution method

Following Auerbach and Kotlikoff (1987), we solve the model by using an improved Gauss-Seidel algorithm[[6]](#footnote-7). Intuitively, the Gauss-Seidel algorithm can be thought of as a Walrasian auction system that allocates resources until an economic equilibrium is reached.

The auctioneer in this system has no control over the interest rate or import prices, since our assumption that Australia is a small-open economy means these are set by the global market. But the auctioneer does set the price of goods and services and the aggregate wage, announcing each to a crowd of agents comprising firms, households, the foreign sector and the government. Based on this announcement, each agent proposes its own response. Firms in each sector determine the level of output and demand for capital, labour and intermediate goods that will maximise their market value. Households determine the labour supply and demand for consumption goods that will maximise their lifetime utility. The foreign sector determines a level of exports given their preferences and the announced prices. And the government determines how to balance its budget and allocate government spending in light of the announced prices. All agents – firms, households, the foreign sector, and the government – submit their intended responses to the auctioneer. The auctioneer reviews these responses and continues to adjust the prices/wage until the goods and labour markets have cleared and the wage is equalised across all sectors. The system is competitive because neither the firms, the households, the foreign sector nor the government can on their own determine the prices/wage announced by the auctioneer. The auction therefore results in a competitive equilibrium.

An outline of the solution method is deferred to Appendix F. The model is implemented using Matlab, a commercial software package that is widely used across industry and academia. Typically, it takes around half an hour to solve the model for the steady states and the transition path.

# Calibration

The model is calibrated to match Australian demographic, economic, government and financial data. Where possible the model’s parameters are estimated directly using Australian data sources or reflect actual policy settings. Otherwise, parameters reflect consensus in the literature.

In Version 1.0 we assume that in the absence of policy change the economy is on the balanced growth path. Working toward that end the growth rates of the model’s exogenous trends (that is, the population and labour augmenting technical progress) are constant. Similarly, exogenous lifecycle profiles for relative skill levels and survival probabilities are time-invariant.

When calibrating the model, it is necessary to make a decision about what historical data period the model is calibrated to. Due to data availability it is not always possible to calibrate all parts of the model to the same historical period. Version 1.0 of the model is designed to approximate the Australian economy around 2015-16, which is a period for which most of the data necessary to calibrate the model is available. Underlying this are the following set of key calibration assumptions:

1. parameters governing the production side of the economy and preferences over different commodities are calibrated to the 2014‑15 Australian Input-Output tables;
2. macroeconomic ratios such as the capital-to-output ratio and the net foreign asset to GDP position and government expenditure and revenue to GDP ratios are generally calibrated to the average over the five-year period to 2015-16;
3. survival probabilities, and therefore life expectancy, are calibrated to 2016-18 levels; and
4. fiscal policy variables reflect policy settings as at 2016-17; and
5. the relative earnings profiles and other household lifecycle variables are calibrated to the data in the 2015-16 Survey of Income and Housing (SIH) published by Australian Bureau of Statistics (ABS).

Fiscal policy variables that are expressed in dollar terms (such as income tax thresholds) are normalised to have the same relationship to GDP per capita as they did in the 2015-16 financial year. Along the balanced growth path, all value policy variables grow at the rate of GDP per capita.

Household

### Demographic structure

#### Population growth

Version 1.0 of OLGA assumes a stationary age distribution of the population and a constant population growth rate. This reflects a constant growth rate of the population of 21-year-olds and time-invariant survival probabilities.

The growth rate of the population of 21-year-oldsis assumed to be 1.5 per cent per annum. Chart 1 plots the model’s conditional and cumulative survival probability by age. Survival probabilities are calculated using ABS Life Tables for Australia (2019a). The conditional survival probability is calculated as one minus the proportion of people dying between age and .

Chart 1: Survival probability by age

This chart shows the model's implied cumulative and conditional survival probability of households by age over their lifetime. Both the conditional and cumulative survival probabilities decreases with age, with significantly more rapid decreases later in life.

Source: Authors’ calculation based on ABS (2019a) data.

Given that the survival probabilities are time-invariant, it follows that the age distribution is also time‑invariant and that the total population grows at the same rate as the population of 21-year-olds (that is, ):



Chart 2 plots the actual and model implied age distribution of the population for the base year. The stationary distribution generated by the model overstates the share of the population under 30, while understating the share of the population between 40 and 70. This reflects the fact that the model does not incorporate immigration and assumes a time-invariant survival rate.

Chart 2: Age distribution of the population

This chart shows the age distribution in the data and compares it with the steady-state distribution implied by the model, using the assumed population growth rate and the survival probabilities depicted in chart 1. The chart shows that the share of the population is smaller for older ages.

Source: Authors’ calculation based on ABS (2019a) and ABS (2020a).

#### Skill and technical progress

The households’ earning ability and implied relative labour efficiency are calibrated using cross-sectional earnings data from SIH (ABS, 2017). For each income quintile, earning ability is estimated using the hourly wage rate, defined as gross labour income divided by total hours worked. To eliminate the effect of extreme outliers, we use the median of data from SIH to estimate the lifecycle profiles. These profiles are then rescaled, so that the population-weighted mean of each lifecycle profile matches the population-weighted mean of data from the SIH for each skill type.

Because cross-sectional data suggests that the earning ability of the lowest quintile is effectively zero, we have assumed it is half that of the second lowest quintile (lower middle). A robust spline smoothing function is then used to provide the smoothed profiles used in the model. Chart 3 shows the fit of the estimated profiles to the actual cross‑sectional earnings data.

The efficiency profiles are normalised, such that the sum of efficiency by age and skill type, weighted by respective population share, equals the level of labour augmenting technical progress:



Consistent with the methodology used in the 2021 Intergenerational Report (Commonwealth of Australia, 2021b), the growth rate of labour‑augmenting technical progressis assumed to be 1.5 per cent per annum. This implies a trend GDP growth rate of around 3 per cent per annum.

Chart 3: Relative labour efficiency by age and skill This chart shows the distribution of efficiency of labour by age and skill in the data and also that of implied by the model. It shows that individuals' efficiency of  labour peaks in the early 40s and decreases afterward. This chart also depicts differences in the efficiency of labour across different skill levels for each age. 

Source: Authors’ calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

#### Preferences

##### **Instantaneous utility**

Following the Dynamic Stochastic General Equilibrium (DSGE) literature (for example, King, Plosser and Rebelo (1988)), we set the coefficient of relative risk aversion for all households. This implies an intertemporal elasticity of substitution that is equal to 1/2. Similar to Fehr (2000) and Kudrna et al. (2015), we have calibratedfor each skill type to match data on lifetime labour supply reported in SIH (ABS, 2017). All these parameters are reported in Table 2 below.

Table 2: Parameter values for the utility function

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Parameter | Low | Lower-middle | Middle | Upper-middle | High | All |
|  |  |  |  |  |  | 0.99 |
|  |  |  |  |  |  | 2 |
|  | 0.5 | 0.6 | 0.65 | 0.7 | 0.75 |  |
|  |  |  | 5 | 10 | 20 |  |
|  |  |  | 0.02 | 0.02 | 0.02 |  |

Consistent with this calibration, we assume that a typical adult Australian, after accounting for public holidays, personal and recreational leave, is available to work 5 days a week, for 45 weeks per year. Per workday, the household is assumed to have 12 hours available for work and leisure. When combined this implies an annual time endowment of 45 x 5 x 12 = 2700 hours.

Charts 4 and 5 suggest our calibration captures labour supply profiles of workers 30 years and older well, while it tends to overstate the labour supply of workers below 30 years of age. There are kinks in the modelled labour supply profiles which are not apparent in the data. These kinks reflect discrete changes in marginal tax rates induced by the progressive personal income tax schedule. Other researchers (see for example, Kudrna and Tran (2018)) avoid this issue by approximating the tax schedule using a smooth income tax function. This approach has not been adopted here because it potentially understates the effects of changes in the progressive tax schedule.

Chart 4: Household labour supply by age and skill

This chart shows the distribution of labour supply in the model and compares it with data. Labour supply of households is shown to peak at around the age of 40 which is then followed by a decline until the retirement. The chart also depicts significant differences in the lifetime labour supply across different skill types.

Source: Authors’ calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Chart 5: Household labour earnings by age and skill This chart shows the distribution of households' labour earnings by age and skill in both data and the model. As explained in the text, labour earnings depend on household labour efficiency, labour supplied and the wage rate. The implication of this is that the profile of labour earnings across ages and skills mimic the profile of labour efficiency and labour supplied. As shown in the chart, and similar to Chart 3 and Chart 4, labour earnings also peaks at around the age of 40 and then gradually decreases until retirement.

Source: Authors’ calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Given, and , there is an implied Frisch elasticity of labour supply:



The relatively low Frisch elasticities generated by the calibration methodology are consistent with the micro-econometric literature surveyed by Keane (2011), which suggests that workers have relatively low elasticities during their prime earning years (see Chart 6).

Chart 6: Implied Frisch elasticities by age and skill

This chart shows the implied (Frisch) elasticity of labour supply implied by the model by age and skill level. The chart shows that the labour supply elasticity troughs at around the age of 35-45 depending on the skill level of households, followed by increase until around the time of retirement. The chart shows that households of higher skill levels generally has lower labour supply elasticities at all ages.

Source: Authors’ calculation.

##### **Household savings and bequests**

The aggregation of individual lifecycle profiles of household savings determines the steady state rate of national saving. The accumulation of household savings is jointly determined by the discount factor and the ‘warm glow’ motive to leave intended bequests.

Following the literature households have a common discount factor assumed to be. Given the calibrated consumption to GDP ratio this ensures the economy’s initial net foreign liability position is consistent with the data at roughly 50 per cent of GDP (see Table 18 below for details).

Following the empirical study of Fink and Redaelli (2005), we have assumed that only households of the high, upper middle and middle skills accumulate assets over their lifetime with the intention of leaving a bequest. For those household types,  is calibrated to match their end-of-life savings in actual data, and is calibrated to have a total bequest-to-GDP ratio of around 3 per cent. While this is only around half the level found by the Productivity Commission (2021), the model currently ignores the value of land used for dwellings which accounts around half of the wealth held by households. This means calibrated annual bequests are consistent with the findings of the Productivity Commission of around 1 per cent of total household wealth. These parameters are reported in Table 2.

As Chart 7 shows, the model’s implied distribution of household wealth does a reasonably good job in tracking the cross-sectional distribution of wealth. Given this the model also matches well the cross-sectional distribution of gross household income (see Chart 8).

Chart 7: Household wealth by age and skillThis chart shows the distribution of wealth by age and skill in the data and also that implied by the model. This chart shows that households wealth increases gradually with age, until it begins to decline as households draw down their savings. Households of higher skill types accumulate wealth for longer, consistent with them working until a later age. 

Source: Authors’ calculations based on SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Chart 8: Gross household income by age and skill

This chart shows the distribution of gross household income by age and skill in the data and also that implied by the model. As explained in the text, household income depends on labour earnins and capital income which is influenced by the saving motives of households in the model. The implication of this is that the gross income of households mimic the behaviour of labour earnings and wealth across ages and skill levels.Source: Authors’ calculations based on SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

### Production sector

Version 1.0 of OLGA includes three goods sectors: Agriculture (AGR), Mining (MIN), and Manufacturing (MAN); and four services sectors: Utilities (UTL), Construction (CST), Services (SRV), and Dwellings (DWE). All sectors produce output that is exported and all sectors except for dwellings faces competing imports. Appendix G shows the concordance with the ABS Input-Output Industry Group.

#### Production technology

##### **Primary factor and intermediate shares**

We use the ABS (2018a) Input-Output (IO) table to calibrate the CES weights for the production and distribution sectors. The Input-Output table provides information about the supply and use of factors and products, and the inter-relationships between sectors. Table 3 recasts the broader 114 sector IO table reported by the ABS into a seven sector version consistent with the sectoral aggregation above.

Table 3: Australian Input-Output table (2014-15) per cent GDP

Source: Authors’ calculation based on ABS (2018a) data.

Producer and import prices are normalised to unity in the initial steady state,for allThe scaling term in the production functionis calibrated to ensure the gross domestic output (GDP) per worker is one in the initial steady state. Given these assumptions the initial steady state prices of intermediate goods and services, the CES weights in the production sector are estimated using the factor input and expenditure cost shares reported in Table 3. Further details on estimation are provided at the end of Appendix C.

Table 4 reports the CES weights for the primary factor and intermediate inputs implied by the purple and red blocks of Table 3.

Table 4: Production weights

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | Consistent with cost share: *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.1590 | 0.1489 | 0.1934 | 0.1502 | 0.2062 | 0.3900 | 0.0000 |
|  | 0.2873 | 0.3951 | 0.1006 | 0.2895 | 0.0977 | 0.1620 | 0.7559 |
|  | 0.5537 | 0.4560 | 0.7060 | 0.5604 | 0.6961 | 0.4480 | 0.2421 |

Source: Authors’ calculation based on ABS (2018a) data.

##### **Factor substitution**

The elasticity of substitution between capital, labour and intermediate inputsis assumed to be 0.5 for all consistent with estimates of the elasticity of substitution between capital and labour reported by Hutchings and Kouparitsas (2012).

##### **Investment/adjustment cost parameters**

The industry capital depreciation rates for the seven sectors respectively, as reported in Table 5 are calibrated based on ABS (2021a).

Table 5: Capital depreciation rates

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.093 | 0.066 | 0.107 | 0.040 | 0.109 | 0.065 | 0.030 |

Source: Authors’ calculation based on ABS (2021a) data.

Following the broader DSGE literature the parameters for capital adjustment costsare set to 2.5 for all sectors. Mendoza and Tesar (1998 and 2005) argue that this value is consistent with the average rate of convergence to the long-run balanced growth path estimated by Barro and Sala-i-Martin (2004).

### Rest of the world

##### **Foreign demand shares and elasticity**

The level of foreign aggregate demand is normalised to be 1 for all sectors. The CES weights for Australian varieties in foreign demandfor all sectors are calibrated using data on global exports from the GTAP Data Base (Aguiar et al. (2019)). This is consistent with the methodology used in multi-country models (for example, Hertel (1997); McKibbin and Wilcoxen (1998)). Taking into account required net export to GDP ratio to stabilise net foreign assets as a ratio to GDP and, the CES weights for all goods in foreign aggregate demandare calibrated to match the distribution of exports reported in blue block of Table 3. Table 6 shows the final calibrated values of  as a per cent of GDP.

Consistent with the elasticity of substitution between different commodities for domestic households and firms, the elasticity of substitution between different types of goodsis set to 0.5. This reflects that consumers are relatively reluctant to substitute between commodities. The elasticities of substitution between varieties of the same good from different exporters are based on those from the GTAP Data Base. These are also presented in Table 6.

Table 6: Export demand parameters

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.082 | 0.265 | 0.038 | 0.028 | 0.001 | 0.017 | 0.001 |
|  | 6.4 | 6.3 | 7.3 | 5.6 | 3.8 | 3.8 | 3.8 |
|  | 2.24% | 8.61% | 6.13% | 0.03% | 0.07% | 5.92% | 0.14% |

Source: Authors’ calculations based on Aguiar et al. (2019) and ABS (2018a). The unit ofis % of GDP.

### Distribution sector

#### Composite goods

##### **Expenditure shares**

The CES weights in the distribution sector composite goods and services functions are calibrated to match final expenditure shares reported in the ABS (2018a) IO table. In particular, the weights for intermediate, consumption, investment, and government consumption are calibrated to match the expenditure shares implied by the purple (intermediate only), green, grey, and orange blocks of Table 3. Further details on calibration are provided at the end of Appendix E. The expenditure shares implied by these data are summarised in Table 7.

The ABS Input-Output tables do not provide investment expenditure by purchasing industry (this means there is only one private investment column in the Input-Output table). To better capture the investment goods purchased by different sectors the investment goods sold by each industry have been allocated across the purchasing industries by also using additional data on private investment by capital type and two-digit industry from the ABS National Accounts 5204.0. This allocation has also been used to allocate the GST paid on investment goods and services to different purchasing industries.

Table 7: Expenditure weights

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | Consistent with cost share: *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.0105 | 0.0037 | 0.1528 | 0.0287 | 0.0010 | 0.5835 | 0.2198 |
|  | 0.2216 | 0.0000 | 0.3971 | 0.0000 | 0.1744 | 0.2069 | 0.0000 |
|  | 0.0000 | 0.0506 | 0.0709 | 0.0000 | 0.7716 | 0.1070 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.3523 | 0.0000 | 0.1997 | 0.4479 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.1678 | 0.0000 | 0.6821 | 0.1501 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.5547 | 0.0000 | 0.1048 | 0.3405 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.3102 | 0.0000 | 0.4160 | 0.2738 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.9153 | 0.0847 | 0.0000 |
|  | 0.3134 | 0.0060 | 0.2476 | 0.0431 | 0.0558 | 0.3342 | 0.0000 |
|  | 0.0044 | 0.1767 | 0.2156 | 0.0341 | 0.1318 | 0.4373 | 0.0000 |
|  | 0.1477 | 0.1827 | 0.3436 | 0.0334 | 0.0112 | 0.2814 | 0.0000 |
|  | 0.0006 | 0.0522 | 0.0821 | 0.4704 | 0.0737 | 0.3209 | 0.0000 |
|  | 0.0025 | 0.0128 | 0.2747 | 0.0095 | 0.4346 | 0.2659 | 0.0000 |
|  | 0.0091 | 0.0115 | 0.1420 | 0.0282 | 0.0481 | 0.7612 | 0.0000 |
|  | 0.0000 | 0.0070 | 0.0253 | 0.0047 | 0.2144 | 0.7486 | 0.0000 |
|  | 0.0014 | 0.0004 | 0.0261 | 0.0036 | 0.0007 | 0.9670 | 0.0008 |

Source: Authors’ calculation based on ABS (2018a) and ABS (2021a) data.

##### **Substitution of varieties**

For the distribution sector, the elasticities of substitution in the basket of goods for the composite consumption, investment, intermediate inputs, and government spending functions () are set to 0.5.

#### Import demand

##### **Import shares**

The allocation of expenditure across domestic and imported alternatives follows the same methodology as the composite goods and services. The added complication is that prices must be adjusted for GST and other ad valorem taxes. The CES weights are derived from the cost shares of imported goods. The cost shares of imported goods are also reported in the ABS (2018a) as a supplement to the IO table. Table 9 reports the seven-sector version of that table, which adopts the same colour coding as Table 3. The expenditure cost shares implied by Tables 3, 8 and 9.

Table 8: Allocation of Imports (per cent of GDP)



Source: Authors’ calculation based on ABS (2018a) data.

Table 9: Import weights

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | Consistent with cost share: *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.0791 | 0.1593 | 0.5050 | 0.0008 | 0.0000 | 0.0433 | 0.0021 |
|  | 0.0200 | 0.000 | 0.7208 | 0.0000 | 0.0050 | 0.0143 | 0.0000 |
|  |
|  |
|  |
|  |
|  |
|  |
|  | 0.0091 | 0.0272 | 0.4463 | 0.0012 | 0.0004 | 0.0250 | 0.0000 |
|  | 0.0152 | 0.0355 | 0.5346 | 0.0006 | 0.0004 | 0.0450 | 0.0000 |
|  | 0.0139 | 0.3427 | 0.4559 | 0.0005 | 0.0010 | 0.0319 | 0.0000 |
|  | 0.0620 | 0.0759 | 0.6278 | 0.0006 | 0.0002 | 0.0262 | 0.0000 |
|  | 0.0288 | 0.0134 | 0.3514 | 0.0010 | 0.0001 | 0.0402 | 0.0000 |
|  | 0.0403 | 0.2858 | 0.4775 | 0.0007 | 0.0003 | 0.0391 | 0.0000 |
|  | 0.0000 | 0.0023 | 0.3177 | 0.0000 | 0.0007 | 0.0107 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.4369 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Source: Authors’ calculation based on ABS (2018a) data.

##### **Import substitution**

The elasticities of substitution between domestic and imported varietiesare consistent with the parameterisation of the GTAP model. In the GTAP model, the elasticity of substitution between domestic and imported varieties for a given commodity is set to half the value of the substitution between varieties of the same commodity from different exporters. We take these elasticities and aggregate them into the sectors in OLGA using each commodity’s weight within that sector. The resulting elasticities are shown in Table 10.

Table 10: Import demand elasticities

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 1.9 | 17.1 | 2.9 | 2.8 | 1.9 | 1.9 | 1.9 |
|  | 2.5 | 3.8 | 3.4 | 2.8 | 1.9 | 1.9 | 1.9 |
|  | 2.2 | 0.9 | 2.0 | 2.8 | 1.9 | 1.9 | 1.9 |

Source: Authors’ calculations based on Aguiar et al. (2019).

### Government

The key aim of the calibration of the government sector in OLGA is to match Commonwealth government revenue, expenditure, and net debt levels. However, on the revenue side the model also incorporates state payroll taxes. In addition, a significant portion of Commonwealth revenue is usually provided to state governments (in particular, GST revenue). Therefore, government expenditure in the model can be better thought of as being calibrated to expenditure funded by Commonwealth revenue (and payroll tax) rather than expenditure directly undertaken by the Commonwealth government.

#### Taxation

Calibrated tax revenues are generally based on the ABS Taxation Revenue (ABS, 2021b) averaged over the 5-year period to 2015-16.

##### **Personal (individuals) income tax**

All marginal rates, thresholds, offsets, and the capital gains discount are calibrated to policy for the 2017-18 financial year (see Table 11 and Chart 9 for further details).

Table 11: Personal income tax schedule

|  |  |  |
| --- | --- | --- |
|  | Income threshold | Marginal tax rate\* |
| J |  |  |
| 0 | 0 | 0 |
| 1 | $18,200 | 0.2\* |
| 2 | $37,000 | 0.345\* |
| 3 | $87,000 | 0.39\* |
| 4 | $180,000 | 0.47\* |

\* Includes the 2 per cent Medicare Levy, except for the first marginal rate which includes half the levy.

The capital gains discount for domestic equity is set to 0.5, consistent with current tax legislation.

The model baseline calibration includes two personal income tax offsets: the Low-Income Tax Offset (LITO); and the Seniors and Pensioners Tax Offset (SAPTO). Table 12 presents the parameters for each of the offsets. Consistent with the legislation, the SAPTO is exhausted before the LITO is applied. While this does not matter for the final tax liability, it matters for determining the actual effective marginal tax rate that an individual household faces. Because the model’s steady state tax policy settings are calibrated to the 2017-18 financial year, we do not include more recent policies such as the low- and middle-income tax offset (LMITO) in the baseline calibration.

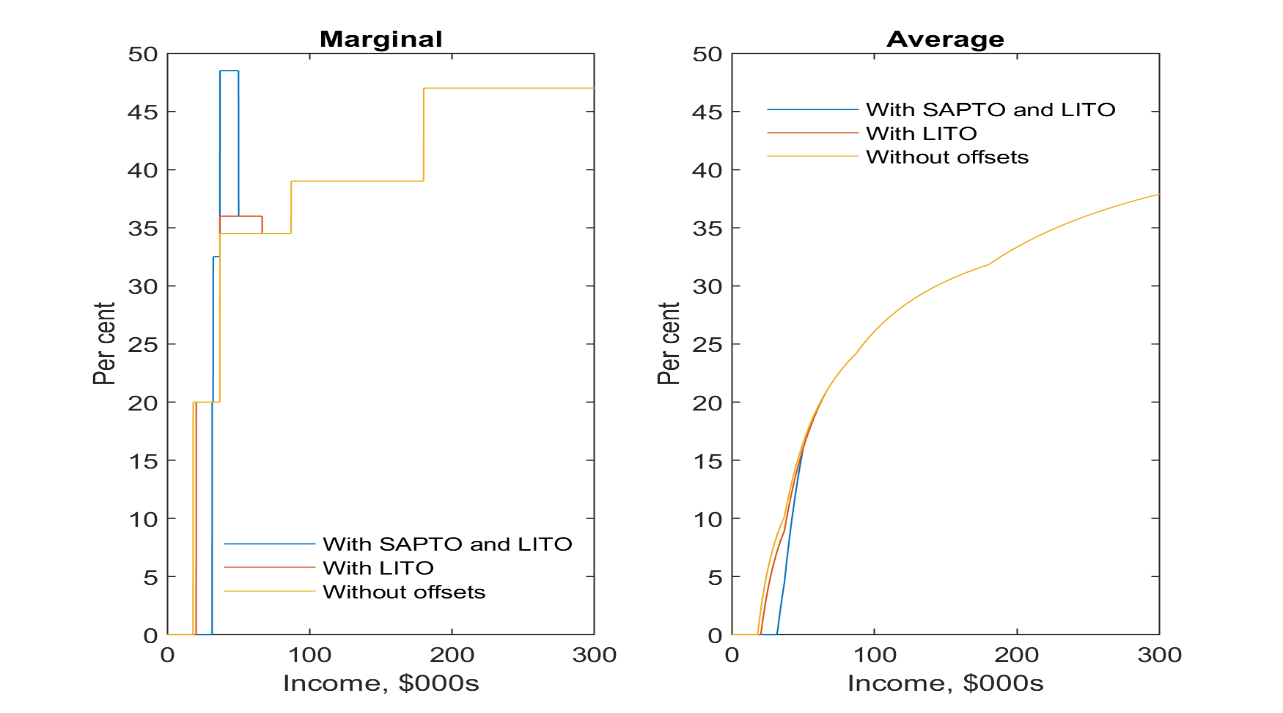
Table 12: Personal income tax offsets

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Offset | Maximum offset | Taper rate | Lower income threshold | Upper income threshold |
|  |  |  |  |  |
| SAPTO | $2230 | 0.125 | $32,279 | $50,119 |
| LITO | $445 | 0.015 | $37,000 | $66,667 |

The tax schedules resulting from the combination of the personal income tax schedule and the personal income tax offsets are shown in Chart 9.

We calibrate the ratio of income tax deductions/exemptionsfor each household type to match the lifecycle profile of tax liabilities (see Chart 10). The deductions/exemption ratio is 0.50 for the lowest skill type, 0.1 for the lower middle and middle skill types; and 0.12 for the upper middle and upper skill types.

Chart 9: Personal income tax rates



Note: The ‘With SAPTO and LITO’ tax schedule applies to those meeting the age pension eligibility age criteria, and the ‘With LITO’ schedule applies to everybody else.

Chart 10: Personal income tax liabilities by age and skill

This chart shows the distribution of household income tax liabilities by age and skill in the data and also that implied by the model. The amount of income tax is closely linked to gross income of households, and therefore shows a similar pattern over the lifetime. The chart shows that households' income tax liabilities vary significantly with age and skill, indicating the progressivity of income tax system and the differences in 

Source: authors’ calculations based on data from SIH (ABS, 2017) and ABS Life Tables (2019a). Note: Broken lines are medians of cross-sectional data relative to the population-weighted mean from the SIH (using the model age distribution), and solid lines are the calibrated profiles relative to the population-weighted mean.

Given the calibration, the average effective personal income tax rateis 19.9 per cent and personal income tax revenue is around 13 per cent of GDP. This is broadly consistent with tax revenue from individual income tax, superannuation fund income tax and fringe benefit tax revenue reported in the ABS Taxation Revenue statistics (ABS 2021b).[[7]](#footnote-8)

##### **Corporate tax**

Version 1.0 of OLGA matches a statutory corporate income tax rate of 30%. Because there is a single representative firm in each sector, we cannot explicitly model the legislated lower tax rates for small and medium-sized businesses.

Under current legislation, interest on corporate debt is fully deductible for CIT purposes. While the model allows for an investment allowance, it is generally not applicable under current legislation so for all sectors.

Conditional on the above assumptions, we jointly calibrate: the share of the modelled production sector taxbase that is taxable under the corporate income tax system , debt to asset ratios; and the proportion of depreciation that can be deducted for corporate income tax purposes, to match each sector’s (based on ATO, 2018b) corporate tax revenue as a share of GDP, debt interest deductions as a share of GDP, and depreciation deductions as a share of GDP using the following iterative approach.

1. Calculate an initial estimate of the incorporated share in each sector based on data on incorporated and unincorporated capital stocks and capital rental prices reported in the ABS Estimates of Industry Multifactor Productivity publication (ABS, 2021d).
2. Calibrate the debt-to-asset ratio required to match interest expenses reported in company tax return data.
3. Calibrate the proportion of depreciation deducted for tax purposes to match those reported in the company tax return data. In practice, firms can only claim depreciation at the historical cost of an asset and because of inflation the full value of depreciation can therefore not be deducted. We estimate that inflation reduces the net present value of depreciation deductions by around 15 per cent, so we limit the proportion of depreciation that can deducted to a maximum value of 0.85.
4. If the result of the above procedure does not match the tax revenue, interest expenses and depreciation deductions as reported in the company tax data, then return to step 1. This process is repeated until the model matches the reported data as closely as possible.

Using this process yields the corporate finance and tax parameters summarised in Table 13.

Table 13: Company tax parameters

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.22 | 0.99 | 0.65 | 0.35 | 0.58 | 0.83 | 0.00 |
|  | 0.44 | 0.43 | 0.70 | 0.65 | 0.60 | 0.29 | 0.30 |
|  | 0 | | | | | | |
|  | 0.71 | 0.85 | 0.27 | 0.85 | 0.29 | 0.41 | 0.00 |
|  | 0.1% | 0.6% | 0.3% | 0.0% | 0.3% | 3.0% | 0.0% |

Source: Authors’ calculation based on ATO (2018b) and ABS (2021a, 2021d) data. The unit of is % of GDP.

Interest withholding tax

The effective withholding tax ratefor non-residents is set to 3 per cent. This ensures interest withholding tax revenue as a percentage of GDP matches the estimate in ABS (2021b) of 0.1 per cent. This parameter could be thought of as the weighted average of statutory rates for all countries with which Australia does or does not have a preferential tax treatment.

##### **Payroll tax**

In reality, payroll tax rates vary by state and only apply to firms above a certain size. However, because we do not model sub-national economies or differentiate firms by size, we model payroll tax as an industry specific effective rate calculated from data on payroll tax paid and compensation of employees (ABS 2018a and 2020b). The effective rates are reported in Table 14 below for all the sectors.

Table 14: Effective payroll tax rates

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.014 | 0.044 | 0.035 | 0.038 | 0.022 | 0.026 | 0.000 |

Source: Authors’ calculation based on ABS (2018a and 2020b) data.

##### **Goods and services tax**

Consistent with current legislation the statutory GST rate is set to 10 per cent. The GST coverage factorsare calibrated based on the ABS (2018a) Input-Output tables, as reported in Table 15 below.

Table 15: Goods and services tax coverage factors

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.0565 | 0.8954 | 0.6829 | 0.6931 | 0.8369 | 0.5523 | 0.0005 |
|  | 0.1202 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.1750 | 0.0000 | 0.0000 | 0.1150 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 1.0000 | 0.5830 | 0.0000 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0007 | 0.0685 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0058 | 0.3682 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0082 | 0.8185 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0116 | 1.0708 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0194 | 0.8764 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0426 | 0.1208 |
|  | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 |

Source: Authors’ calculation based on ABS (2018a) and ABS (2020a) data.

The GST coverage factors for investment goods by sector have been disaggregated across different purchasing sectors using the same approach as that used to allocate investment goods from different supplying sectors to purchasing sectors.

##### **Other indirect taxes**

In a similar vein to the GST coverage factors, effective ad valorem rates for other indirect taxes such as duties and subsidies for various expenditure bundlesare calibrated based on the ABS (2018a) Input-Output table, as reported in Table 16 below.

Table 16: Effective indirect tax rates

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Parameter | *j* = | | | | | | |
| AGR | MIN | MAN | UTL | CST | SRV | DWE |
|  | 0.0030 | 0.0051 | 0.0422 | -0.0004 | 0.0000 | 0.0118 | 0.0000 |
|  | 0.0016 | | | | | | |
|  | 0.0000 | | | | | | |
|  | 0.0112 | | | | | | |
|  | 0.0000 | | | | | | |
|  | 0.0701 | | | | | | |
|  | 0.0125 | | | | | | |
|  | 0.0000 | | | | | | |
|  | 0.0030 | 0.0002 | 0.0060 | 0.0014 | 0.0017 | 0.0046 | 0.0000 |
|  | 0.0051 | 0.0053 | 0.0152 | 0.0174 | 0.0104 | 0.0241 | 0.01188 |
|  | 0.0422 | 0.0278 | 0.0103 | 0.0413 | 0.0080 | 0.0359 | 0.0059 |
|  | -0.0004 | -0.0086 | 0.0006 | 0.0003 | 0.000 | -0.0065 | 0.0005 |
|  | 0.0000 | -0.0014 | 0.0076 | 0.0002 | 0.0000 | 0.0000 | 0.0000 |
|  | 0.0118 | -0.0128 | 0.0045 | 0.0061 | 0.0042 | 0.0046 | 0.0110 |
|  | 0.000 | 0.0000 | 0.000 | 0.0000 | 0.000 | 0.0000 | 0.0000 |

Source: Authors’ calculation based on ABS (2018a) data.

#### Spending

The ratio of government spending to GDP in the initial steady stateis the level necessary to ensure a stable government debt to GDP ratio. This is calculated as modelled government revenue less modelled government transfers and the debt stabilising primary balance. The implied government spending to GDP ratio is lower than suggested in the data (ABS, 2021e) because there are several state and local government taxes that are not captured in the model and some government transfers are modelled to be larger share of GDP than reported in the data.

#### Transfers

##### **Age pension**

The age pension function parameters including, age eligibility, maximum pension rate, income threshold, income test taper rate, asset threshold, and asset test taper rate, match actual values as of December 2017 (see Table 17).

Under the current system, an individual’s age pension depends on their relationship status, with single payments higher than for those in a couple. We calculate a maximum age pension payment by weighting the rates for single and couple recipients by the proportion of each type of recipients reported by the Department of Social Services (2018). The maximum age pension payment in the model is calibrated to be the same as of December 2017 by adjusting the replacement ratio parameter.

Consistent with the calculation of the maximum age pension payment, the income and asset test thresholds are weighted averages of the single and couple thresholds[[8]](#footnote-9).

Table 17: Age pension parameters

|  |  |  |
| --- | --- | --- |
| Parameter | Variable | Value |
| Eligibility age |  | 66 |
| Maximum benefit-Replacement ratio |  | 0.26 |
| Maximum age pension payment | pmax | $20,414.94 |
| Asset test-lower bound (max benefit) |  | $366,379 |
| Asset test-upper bound (no benefit) |  | $621,566 |
| Asset test-taper rate |  | 0.08 |
| Income test-lower bound (max benefit) |  | $4,110 |
| Income test-upper bound (no benefit) |  | $44,940 |
| Income test-taper rate |  | 0.5 |
| Deemed income-threshold |  | $50,200 |
| Deemed income-deeming rate below threshold |  | 0.0175 |
| Deemed income-deeming rate above threshold |  | 0.0325 |

##### **Other transfers**

Other social transfers are calculated from SIH (ABS, 2017). These data indicate that individuals in the bottom income quintile who are close to age pension eligibility generally receive some other transfer payment of similar value. In contrast, the other social transfer payments to the top three income quintiles are generally low and we set them to zero at all ages. In light of this, transfers payments received by the lowest household skill type in the year before they are eligible for age pension eligibility is equal to the maximum age pension payment. Total other social transfers are around 5 per cent of GDP, consistent with personal benefit payments less age pension expenditure reported in government budget outcomes (Commonwealth of Australia, 2016).

#### Government’s net debt

The initial steady state government net debt to GDP ratio is calibrated to be 20 per cent of GDP. This is consistent with the level of the Commonwealth government’s net debt before the beginning of the COVID-19 pandemic (Commonwealth of Australia, 2021a).

### Financial sector

As noted above the production and hence financial sector calibration in Version 1.0 of OLGA ignores non-produced assets such as land by only including produced assets.

As previously discussed, we assume that firms maintain fixed debt to equity ratios while households hold constant shares of domestic debt and equity assets, and foreign assets. Given these assumptions, the calibration of corporate taxes and finance in the incorporated sector of the economy (described above) determines the overall debt-to-equity ratio in the economy. The resulting aggregate debt-to-equity ratio in the model is 0.57, which is close to the 0.63 implied by the ABS Australian National Accounts: Finance and Wealth publication (ABS, 2021c).

Subject to this estimate, the remaining rates of return, ratios and shares are calibrated to match historical estimates of the following key financial variables:

1. Consistent with ABS (2021a) the capital stock to GDP ratio is around 3.2. The global benchmark required rate of return is adjusted to target this ratio;
2. Consistent with the ABS (2021c) the foreign ownership share of corporate bonds is around 0.85;
3. Consistent with ABS (2021c) the foreign ownership share of equity is around 0.37. Additional data are used to calibrate the split between capital gains and dividends, with all earnings accruing to foreign investors are returned as a capital gains;
4. Consistent with ATO (2018b), the domestic split between earnings returned as a capital gain or dividend is calibrated to achieve an initial franking credit share of GDP of roughly one per cent;
5. As noted above, consistent with Commonwealth of Australia (2021a) government debt is calibrated to 20 per cent of GDP;
6. Consistent with Australian Office of Financial Management (AOFM (2019)) the foreign ownership share of government debt is around 0.55;
7. The discount factoris adjusted so that the the net foreign liability to GDP ratio is consistent with ABS (2021f). A drawback of this approach is that household foreign investment is modelled as the net rather than gross position. Because the current version of the model ignores the asset value of residential land this approach understates the gross foreign liabilities held abroad by Australian households.

Table 18 shows the ownership of assets in the steady state calibration. In general, these match the financial ratios reported above well.

Table 18: Ownership of assets (per cent of GDP)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Debtor  Creditor | Households | Firms | | | | | Government | Total Domestic | Total Foreign | Gross Assets |
| Bonds | Equity | | | Total |
| Dividends | Capital gain | Total |
| Households |  | 37 (2) | 143 (4) | 7 (4) | 150 (3) | 187 | 8 (6) | 195 | 102 (7) | 298 |
| Firms |  |  |  |  |  |  |  |  |  |  |
| Government |  |  |  |  |  |  |  |  |  |  |
| Foreign |  | 82 (2) |  | 59 (3) | 59 (3) | 141 | 12 (6) | 153 |  |  |
| Gross Liabilities |  | 119 | 143 | 66 | 209 | 328 (1) | 20 (5) | 348 |  | -50\* (7) |

Source: Authors’ calculation. Note:\*denotes Australia’s net foreign asset position.

##### **Household investment**

The domestic and foreign ownership shares in Table 18 imply the share of household savings allocated to non-dwelling assets, which include domestic equity and corporate bonds, government bonds and foreign assets, reported in Table 19.

The share of household savings allocated to dwellings in the initial steady stateis estimated to be 0.27. As noted above, this share ensures that domestic households own all equity in the dwellings sector. Because Version 1.0 of OLGA does not include non-produced factors such as land, only captures the value of dwelling structures (produced assets). This assumption will be relaxed in a development module dedicated to modelling the taxation of housing

Table 19: Allocation of household savings in non-dwellings investment

|  |  |  |  |
| --- | --- | --- | --- |
| Debtor | Asset type | Portfolio weight | |
| Firms | Total |  | 0.49 |
| Bonds |  | 0.17 |
| Equity Dividend |  | 0.29 |
| Equity Capital gain |  | 0.03 |
| Government | Bonds |  | 0.04 |
| Foreign portfolio | Total |  | 0.47 |
| Total |  |  | 1.00 |

Source: Authors’ calculation.

##### **Foreign investment**

Foreign ownership shares of Australian domestic equity, corporate bonds and government bonds are estimated as a residual. The implied initial portfolio weights are reported in Table 20.

Table 20: Initial allocation of foreign investment

|  |  |  |  |
| --- | --- | --- | --- |
| Debtor | Asset type | Portfolio weight | |
| Firms | Total |  | 0.92 |
| Bonds |  | 0.54 |
| Equity Dividend |  | 0.00 |
| Equity Capital gain |  | 0.39 |
| Government | Bonds |  | 0.08 |
| Total |  |  | 1.00 |

Source: Authors’ calculation.

##### **Risk free rate and risk premia**

The small open economy framework implies that the domestic interest rate is exogenous and set by the ROW. In Version 1.0 the global required after tax rate of return for sovereign/government debtis assumed to be 3.0 per cent.

The before-tax rates of return of all financial assets are linked to global required after-tax rate of return with different risk premia applied (see Table 21). These premia are calibrated to match observed relative rates of return on different assets (AMP Capital (2017)).

Table 21: Risk premia and global required return (per cent)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Debtor | Asset type | Premium | | Global required rate of return | |
| Foreign | Risk free bonds |  |  |  | 3.00 |
|  | Portfolio |  | 1.5 |  | 4.50 |
| Government | Bonds |  | 1.00 |  | 4.12\* |
| Firms | Bonds |  | 1.50 |  | 4.64\* |
|  | Equity |  | 2.40 |  | 5.4 |

Note:\*include adjustment for withholding tax. Source: Authors’ calculation based on AMP Capital (2017) data.

### Model benchmark and performance

A summary of the initial steady state macroeconomic variables from the model is reported in Table 22. Overall, these numbers are well in line with the key Australian macroeconomic variables averaged over the 5-year period to 2015-16.

On the income side, the shares of labour and capital income are very close to their actual values. Total capital stock and hours worked line up well with the observed data, suggesting that returns on capital and wages are finely calibrated. The revenue to GDP ratios for major taxes (such as personal income tax, company income tax, and goods and services tax) also generally match the reported data.

On the expenditure side, because the model requires a positive trade balance to stabilise its net foreign liabilities position, we cannot match the negative trade balance in actual data. As such, the model-generated private consumption share of GDP is slightly lower than that observed in Australia. The model also generates a slightly higher investment to GDP ratio. This is a by-product of ignoring fixed factors that do not depreciate and overstating the required investment to cover depreciation.

In terms of government taxation and spending statistics, the model generally matches data well with a few exceptions. First, the model generates higher age pension expenditure to GDP ratio. This is partly because the model has a larger share of the population over 60 than observed in the data. Second, the model misses a fraction of indirect taxes (such as state and local government stamp duty and land tax), and this leads to lower government tax revenue than the actual data. This is partly offset by personal income tax revenue being higher than in the data. As discussed above, the model generates lower government spending to satisfy the government’s budget constraint.

Under these macroeconomic settings, the model does a good job in matching the distribution of Australian household income and wealth measured by Gini coefficients.

Table 22: Initial steady state variables

|  |  |  |
| --- | --- | --- |
| Variable | Data value | Model value |
| Factor income (% of GDP) |  |  |
| Labour income | 53.5 | 54.5 |
| Capital income | 46.5 | 45.5 |
| Factor inputs |  |  |
| Capital stock (% of GDP) | 321 | 328 |
| Hours worked (billion hours) | 20.5 | 20.3 |
| National accounts (% of GDP) |  |  |
| Consumption | 56.3 | 55.3 |
| Investment | 26.7 | 28.5 |
| Government spending | 18 | 14.5 |
| Exports | 20.3 | 23.4 |
| Imports | 21.5 | 21.5 |
| Net exports | -1.2 | 1.8 |
| Govt. Revenue (% of GDP) |  |  |
| Personal income tax | 11.3 | 13.1 |
| Company income tax | 4.3 | 4.4 |
| Goods and services tax | 3.4 | 3.3 |
| Interest withholding tax | 0.1 | 0.1 |
| Payroll tax | 1.3 | 1.4 |
| Other indirect taxes | 3.3 | 2.5 |
| Govt. Payments (% of GDP) |  |  |
| Franking credits | 1.0 | 1.1 |
| Age pension expenditure | 2.5 | 4.1 |
| Other government benefits | 5.3 | 5.0 |
| Government net debt (% of GDP) | 20 | 20.0 |
| Net foreign liabilities (% of GDP) | 54.7 | 50.3 |
| Net income shares by income quintile |  |  |
| Low | 0.08 | 0.08 |
| Lower middle | 0.13 | 0.11 |
| Middle | 0.17 | 0.14 |
| Upper middle | 0.23 | 0.22 |
| High | 0.40 | 0.44 |
| Income Gini Coefficient | 0.32 | 0.35 |
| Wealth shares by income quintile |  |  |
| Low | 0.01 | 0.05 |
| Lower middle | 0.05 | 0.11 |
| Middle | 0.11 | 0.15 |
| Upper middle | 0.21 | 0.21 |
| High | 0.62 | 0.49 |
| Wealth Gini Coefficient | 0.60 | 0.60 |

Source: Authors’ calculation based on ABS (2019b, 2021a, 2021b, 2021c, 2021e, 2021f), ATO (2018b) and Commonwealth of Australia (2016).

# Welfare analysis for policy evaluation

As noted at the beginning of this paper, OLGA like other overlapping generations models has several desirable features which means they are well suited to fiscal policy analysis. Households in OLGA are rational and forward-looking. They have explicit objective functions that are consistent with their expectation of the future path of policy and associated macroeconomic aggregates. This allows an explicit comparison of household welfare across alternative fiscal policies. Moreover, households in OLGA are heterogeneous in their age, skill types, income, and wealth. As such, we can move beyond the macroeconomic aggregates and undertake an individual assessment of each household. This allows Treasury to investigate the distributional and intergenerational effect of fiscal policies.

Following Fehr and Kindermann (2018), and Kudrna and Tran (2018), we measure welfare gains and losses for a household associated with a policy change using a dynamic version of the Hicksian Equivalent Variation (HEV) approach. This approach essentially measures the change in lifetime utility (measured in terms of initial consumption prices and wages) that would leave the household indifferent to the change in policy. If the policy change increases their lifetime utility, the HEV will be positive, and the household will be deemed to be better off.

Let be the age of the household when the policy change is announced at time,be the household’s level of consumption and leisure assuming no policy change (which is typically the steady state path) for their remaining life andbe the household’s level of consumption and leisure under the policy change for their remaining life. Note that this includes households not yet born (that is,). We estimate the HEV via  which is the permanent change in the household’s consumption and leisure over their remaining lifetime sufficient to yield the same utility as under the policy change:



Ifis positive the household is better off under the policy change. Alternatively, ifis negative the household is worse off under the policy change. This approach is similar to the ‘consumption equivalence’ measure adopted by Nishiyama and Reichling (2015).

# Conclusion

The overlapping generations model of the Australian economy (OLGA) described in this paper reflects a significant capability in both physical and human capital. This capability is intended to meet the needs of all Treasury’s stakeholders today and well into the future. With tools such as OLGA, Treasury’s modelling capability is now on par with recognised international fiscal agencies such as the US CBO and JCT.

OLGA is intended to support Treasury’s economy-wide counterfactual fiscal policy analysis. It provides a rigorous tool to quantify the general equilibrium effects of fiscal policy proposals. By including the indirect effects of a policy proposal, OLGA provides a comprehensive assessment of the so-called dynamic cost of a policy proposal. It captures the macroeconomic effects of how a proposal is financed. And it helps Treasury better understand who ends up gaining or losing from the policy – that is, how it affects the welfare of different types of households by age, income, and wealth.

The version of OLGA presented in this paper is referred to as Version 1.0. This version of the model essentially encompasses all the functionality of Treasury’s previous fiscal models. But it overcomes several of the limitations of earlier models. OLGA Version 1.0 has also been designed so that it can be enhanced through discrete development modules. To deliver on the needs of stakeholders, Treasury will continue to develop the model to meet specific needs.

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# Appendix A: An overview of OLGA

OLGA is a dynamic general equilibrium model with heterogeneous demographic structure and multiple production sectors. There are 75 overlapping generations of households in the model. Households are assumed to be uniformly distributed across five skill types. Households provide labour and savings to the production sectors. Firms in the production sectors employ labour, capital, and intermediate inputs to produce final and intermediate goods and services for domestic use or export. Domestically produced goods compete with differentiated goods and services supplied by foreign producers. Firms manage capital investment decisions, and source funds via a notional funds manager from households and foreign investors. There is a government that collects revenue from households and firms via taxes on production, income, and expenditure. This tax revenue finances government spending. The government relies on funds sourced from domestic and foreign investors to meet temporary primary deficits. The households also own foreign financial assets.

Figure A1:   
Overview of the flow of goods/services, factors of production and funds in OLGA

This is a graphic representation of the interactions between the different agents in the model, as described in the text above.

# Appendix B: Households’ optimisation problem

Bellman equation

As noted in Section 2, a household of skill type, who enters the model at the beginning of year , chooses consumption, labour supply and savings to maximise lifetime utility:



where:is the household’s discount factor; , , and  are the household’s consumption, leisure, and beginning-of-period savings at year. This is subject to the intertemporal budget constraint:



By the principle of optimality (Sundaram 2009, Chapter 11), the household’s maximisation problem in period can be recast with the following Bellman equation:



with the intertemporal budget constraint:



Here  is called the value function of the household’s maximisation problem, and is the reward function for the household’s choice of given initial savings of . Intuitively,  is the optimal level of lifetime utility the household can obtain given initial savings in period ; and  is the continuation value for the household with initial savings in period . This alternative representation using Bellman equation demonstrates the household’s recursive problem over time.

Solution technique

Because the household has a finite lifetime, we have:







We can use this information to solve the household’s maximisation problem by backward induction. Specifically, we have the Bellman equation for a household who is 95 years old in period :



Given the functional forms of , the reward function  is concave in . Nevertheless, due to progressivity in personal income tax, means-tested pension and other discontinuous policies, there is no guarantee that is concave in labour supply. Therefore, we can only use the Karush–Kuhn–Tucker conditions to find the optimal solution for , but rely on grid search for .

We proceed by discretising the household’s labour supply. Conditioned on  for some , the optimal level of consumption  and savings  can be solved through the following Lagrangian:



The Karush–Kuhn–Tucker conditions imply:





Together with the complementary slackness conditions:







By substitution, we have:



The complementarity slackness conditions suggest that either  or . Therefore, we can proceed by trial-and-error. First, we assume, and solve for  that satisfies equation . We then check if . If so, we obtain the solution ; otherwise, we let  as the complementarity slackness condition suggests. Given , we can then use the household’s intertemporal budget constraint to obtain .

Proceeding as prescribed above along the grid of labour supply , we obtain a set of candidate solutions  to the Bellman equation given initial savings . The optimal solution  is one that maximises the value of the Bellman equation.

Now knowing  and  given initial savings , we can move backward to solve the household’s Bellman equation at age 94 in period :



Following Stachurski (2011, Chapter 5), one can prove that the reward function  is concave in . But again, there is no guarantee that  is concave in labour supply .

Conditioned on , we have the following Lagrangian:



The Karush–Kuhn–Tucker conditions for optimality again imply:





with the complementary slackness conditions:







The envelop theorem also suggests:



By substitution, we have:

 We solve this equation with the intertemporal budget constraint and complementarity slackness conditions in a similar way as discussed above to obtain the optimal solutions to the household’s Bellman equation at age 94 in period . After this, we go further backward and repeat the process until we derive the optimal solutions for any initial savings from age 21 to 95.

Knowing the optimal decision rules, we can finally work forward to derive the household’s optimal life-time labour supply, consumption, and savings, given initial savings at age 21.

# Appendix C: Firm’s optimisation problem

As noted in Section 2, the objective for the representative firm in each sector is to choose the level of production (that is, the level of capital, labour and intermediate inputs) to maximise the market value of equity:



This is subject to the debt-to-capital ratio:



earnings before interest, tax and amortisation:



company income tax:



budget constraint:



law of motion for capital:



and production technology:



First order conditions

Dropping the subscript for a production sector, the Lagrangian for the representative firm’s problem is:

Here ,, , ,  and  denote the shadow prices of the constraints. The first order conditions of the above problem therefore imply:

















or equivalently:



And we finally have:



This is the no-arbitrage condition for investment which links current capital shadow price  to its future counterpart.

#### Market value of equity

We claim that if



Then



To see this, following Obstfeld and Rogoff (1996, Chapter 2), we multiply both sides of equation by  and substitute for  and , which gives:



Subtracting  from both sides of the equation above and substituting for  on the right-hand side, we further have:



Now using the law of motion for capital as per equation , we have:



Substituting for , we have:



Substituting for , we have:



Further algebra will then give:



Because the production is homogenous of degree one, by Euler’s theorem and the first order conditions for labour and intermediate inputs, we have:



This implies:



Using the firm’s budget constraint, the equation above can be further rewritten as:



By forward iteration, we have replicated equation as follows:



Steady state conditions

At the steady state, we have:







Manipulating the last equation above gives the following:



Letting  be firm’s effective cost of capital net of tax deduction:



We have:



This implies that at the steady state, the marginal return on capital after company income tax is equal to the effective cost of capital plus depreciation.

Using the first second conditions, we also have:







These equations suggest the following relationship between the CES weights and other variables:







It is worth noting that, ,  and  above are the cost shares of capital, labour and intermediate inputs. Assuming that the Australian economy is at steady state, we can then use equations (84) to (86) and the cost share information from Table 4 to estimate the CES weights.

# Appendix D: Modelling foreign import demand (aka Demand for Australian exports)

Consider a representative consumer living in foreign country who solves a nested utility maximising problem. At the top of the nest is the choice between different types of consumption goods (for example iron ore, computers, and foods). Following this is the choice between different varieties of the same consumption good that are produced in different exporting countries (for example Australian iron ore and Brazilian iron ore).

Figure A2: Foreign consumer's nested preference This figure shows the nested CES structure that describes how aggregate foreign consumption is a composite of different commodities, each which in turn in is a composite of varieties from different countries. 

Assuming the foreign consumer’s preferences at the top tier are captured by a constant elasticity of substitution utility function (CES) preference, the problem can be summarised by the following:



subject to:



where:is the elasticity of substitution between different consumption goods,is a weighting parameter with, is the aggregate consumption level,is the aggregate consumption price, and are the price and quantity of good.

The first order condition for optimality implies:



where the price of aggregate consumptionis a function of individual goods prices and their weights in the consumption bundle:



Once a decision is made at the top level, the representative consumer will move to the second level, which involves the choice between varieties of the same goods that are produced in different countries, subject to the value of aggregate expenditure decided at the previous level.

At the second level, the representative consumer has the following maximisation problem:



subject to:



whereis the elasticity of substitution between varieties of the same good that are produced in different countries, is a weighting parameter with , andare the price and quantity of varietyexported from country.

The first order condition for optimality in turn implies:



where the price of the consumption good  is a function of the prices of different varieties and their weights in the variety bundle:



As in Imbs and Méjean (2010), we can combine equations and to have:



From this we can derive the demand for Australian exports as in equation of the paper.

# Appendix E: Distribution sector’s optimisation problem

As discussed in Section 2 of the paper, a notional domestic distribution sector combines imported goods and domestically produced goods to form composite consumption, investment, intermediate goods, and government spending.

Consumption of goods and services

For each household, the distribution sector first chooses the consumption bundle that maximises the value of composite consumption:



subject to the allocated budget:



Following Obstfeld and Rogoff (1996, Chapter 4), we can obtain the optimal demand for each good:



and the price index for the composite consumption good:



The distribution sector then chooses from a basket of goods composed of domestically producedand importedvarieties, to maximise the value of consumption:



subject to the allocated expenditure:



The optimal demand for domestic and imported goods is again as follows:





And the price index for composite consumption good is:



Investment goods and services

For capital formation of sector, the distribution sector first chooses the investment goods bundle to maximise the value of the firm’s gross fixed capital expenditure:



subject to the budget constraint:



The optimal demand and implied price index functions are as follows:





The distribution sector then chooses from a variety of domestic and imported investment goods to maximise the value of investment:



subject to the allocated budget:



The optimal demand for domestic and imported goods is as follows:







Intermediate goods and services

Similarly, for intermediate input of sector, the distribution sector first chooses the intermediate goods bundle to maximise the value of the firm’s expenditure:



subject to the budget constraint:



The optimal demand and implied price index functions are as follows:





The distribution sector then chooses from a variety of domestic and imported intermediate goods to maximise the value of the firm’s use of intermediate goods:



subject to the allocated budget:



The optimal demand and implied price index functions are as follows:







Government purchases of goods and services

As discussed in the paper, the government is assumed to consume a fixed bundle of goods and servicessupplied by the distribution sector. Given the market price of each good, total government spending is:



and aggregate real government expenditure is:



The distribution sector then chooses from a variety of domestic and imported goods to maximise the value of the government’s purchase of goods:



subject to the budget constraint:



The optimal demand and implied price index functions are as follows:







Calibration of CES weights

The optimal demand conditions we have derived above for the distribution sector suggest the following relationship between the CES weights and other variables:









Here , ,  and  are the cost share of each good in consumption, government spending, investment, and intermediate use implied by Table 3 of the paper. We also have:









Here

,

,

 and



are the cost shares of imports in consumption, government spending, investment, and intermediate use as implied by Table 3 and

Table 8 of the paper.

As these relations hold for all periods including the base year of the model, we can then use equations (86) to (93) and the cost share information from Table 3 and

Table 8 to estimate the CES weights .

# Appendix F: Gauss-Seidel algorithm for solving OLGA

Following Auerbach and Kotlikoff (1987), we solve the Gauss-Seidel algorithm to solve the model. Specifically, the computation of OLGA’s equilibrium responses to a policy scenario has three stages:

1. Solving for the long-run steady state of the economy before the assumed change in policy occurs;
2. Solving for the long-run steady state which the economy eventually converges to after the policy takes effect; and
3. Solving for the transition path that the economy takes between these two steady states.

Each stage involves the following six steps:

Step 1: Start with an initial estimate for the average wage level and government policies, and each sector’s labour input or output level and goods prices.

Step 2: Solve the firm’s problem for each sector given labour input (or output level) and goods prices. This determines each sector’s wage, output and demand for capital, labour and intermediate goods.

Step 3: Solve each household’s problem given the average wage and goods prices. This determines labour supplied, and demand for consumption goods.

Step 4: Solve export and government demand for each sector given goods prices, and calculate the government’s total tax revenue and expenditure.

Step 5: Check if all goods markets clear, if wage is equalised across all sectors, if aggregate labour inputs equals labour supplied, and if the government’s budget constraint holds.

Step 6: If Step 5 is not passed, update the estimate using the quasi-Newton method as suggested in Ludwig (2007) and Heer and Maussner (2009); and return to Step 2.

# Appendix G: Sectoral concordance between OLGA and ABS Input-Output Industry Group (IOIG)

|  |  |  |  |
| --- | --- | --- | --- |
| IOIG | Description | OLGA Sector | Code |
| 0101 | Sheep, Grains, Beef and Dairy Cattle | Agriculture | AGR |
| 0102 | Poultry and Other Livestock | Agriculture | AGR |
| 0103 | Other Agriculture | Agriculture | AGR |
| 0201 | Aquaculture | Agriculture | AGR |
| 0301 | Forestry and Logging | Agriculture | AGR |
| 0401 | Fishing, Hunting and Trapping | Agriculture | AGR |
| 0501 | Agriculture, Forestry and Fishing Support Services | Agriculture | AGR |
| 0601 | Coal Mining | Mining | MIN |
| 0701 | Oil and Gas Extraction | Mining | MIN |
| 0801 | Iron Ore Mining | Mining | MIN |
| 0802 | Non Ferrous Metal Ore Mining | Mining | MIN |
| 0901 | Non Metallic Mineral Mining | Mining | MIN |
| 1001 | Exploration and Mining Support Services | Mining | MIN |
| 1101 | Meat and Meat product Manufacturing | Manufacturing | MAN |
| 1102 | Processed Seafood Manufacturing | Manufacturing | MAN |
| 1103 | Dairy Product Manufacturing | Manufacturing | MAN |
| 1104 | Fruit and Vegetable Product Manufacturing | Manufacturing | MAN |
| 1105 | Oils and Fats Manufacturing | Manufacturing | MAN |
| 1106 | Grain Mill and Cereal Product Manufacturing | Manufacturing | MAN |
| 1107 | Bakery Product Manufacturing | Manufacturing | MAN |
| 1108 | Sugar and Confectionery Manufacturing | Manufacturing | MAN |
| 1109 | Other Food Product Manufacturing | Manufacturing | MAN |
| 1201 | Soft Drinks, Cordials and Syrup Manufacturing | Manufacturing | MAN |
| 1202 | Beer Manufacturing | Manufacturing | MAN |
| 1205 | Wine, Spirits and Tobacco | Manufacturing | MAN |
| 1301 | Textile Manufacturing | Manufacturing | MAN |
| 1302 | Tanned Leather, Dressed Fur and Leather Product Manufacturing | Manufacturing | MAN |
| 1303 | Textile Product Manufacturing | Manufacturing | MAN |
| 1304 | Knitted Product Manufacturing | Manufacturing | MAN |
| 1305 | Clothing Manufacturing | Manufacturing | MAN |
| 1306 | Footwear Manufacturing | Manufacturing | MAN |
| 1401 | Sawmill Product Manufacturing | Manufacturing | MAN |
| 1402 | Other Wood Product Manufacturing | Manufacturing | MAN |
| 1501 | Pulp, Paper and Paperboard Manufacturing | Manufacturing | MAN |
| 1502 | Paper Stationery and Other Converted Paper Product Manufacturing | Manufacturing | MAN |
| 1601 | Printing (including the reproduction of recorded media) | Manufacturing | MAN |
| 1701 | Petroleum and Coal Product Manufacturing | Manufacturing | MAN |
| 1801 | Human Pharmaceutical and Medicinal Product Manufacturing | Manufacturing | MAN |
| 1802 | Veterinary Pharmaceutical and Medicinal Product Manufacturing | Manufacturing | MAN |
| 1803 | Basic Chemical Manufacturing | Manufacturing | MAN |
| 1804 | Cleaning Compounds and Toiletry Preparation Manufacturing | Manufacturing | MAN |
| 1901 | Polymer Product Manufacturing | Manufacturing | MAN |
| 1902 | Natural Rubber Product Manufacturing | Manufacturing | MAN |
| 2001 | Glass and Glass Product Manufacturing | Manufacturing | MAN |
| 2002 | Ceramic Product Manufacturing | Manufacturing | MAN |
| 2003 | Cement, Lime and Ready-Mixed Concrete Manufacturing | Manufacturing | MAN |
| 2004 | Plaster and Concrete Product Manufacturing | Manufacturing | MAN |
| 2005 | Other Non-Metallic Mineral Product Manufacturing | Manufacturing | MAN |
| 2101 | Iron and Steel Manufacturing | Manufacturing | MAN |
| 2102 | Basic Non-Ferrous Metal Manufacturing | Manufacturing | MAN |
| 2201 | Forged Iron and Steel Product Manufacturing | Manufacturing | MAN |
| 2202 | Structural Metal Product Manufacturing | Manufacturing | MAN |
| 2203 | Metal Containers and Other Sheet Metal Product manufacturing | Manufacturing | MAN |
| 2204 | Other Fabricated Metal Product manufacturing | Manufacturing | MAN |
| 2301 | Motor Vehicles and Parts; Other Transport Equipment manufacturing | Manufacturing | MAN |
| 2302 | Ships and Boat Manufacturing | Manufacturing | MAN |
| 2303 | Railway Rolling Stock Manufacturing | Manufacturing | MAN |
| 2304 | Aircraft Manufacturing | Manufacturing | MAN |
| 2401 | Professional, Scientific, Computer and Electronic Equipment Manufacturing | Manufacturing | MAN |
| 2403 | Electrical Equipment Manufacturing | Manufacturing | MAN |
| 2404 | Domestic Appliance Manufacturing | Manufacturing | MAN |
| 2405 | Specialised and other Machinery and Equipment Manufacturing | Manufacturing | MAN |
| 2501 | Furniture Manufacturing | Manufacturing | MAN |
| 2502 | Other Manufactured Products | Manufacturing | MAN |
| 2601 | Electricity Generation | Utilities | UTL |
| 2605 | Electricity Transmission, Distribution, On Selling and Electricity Market Operation | Utilities | UTL |
| 2701 | Gas Supply | Utilities | UTL |
| 2801 | Water Supply, Sewerage and Drainage Services | Utilities | UTL |
| 2901 | Waste Collection, Treatment and Disposal Services | Utilities | UTL |
| 3001 | Residential Building Construction | Construction | CST |
| 3002 | Non-Residential Building Construction | Construction | CST |
| 3101 | Heavy and Civil Engineering Construction | Construction | CST |
| 3201 | Construction Services | Construction | CST |
| 3301 | Wholesale Trade | Services | SRV |
| 3901 | Retail Trade | Services | SRV |
| 4401 | Accommodation | Services | SRV |
| 4501 | Food and Beverage Services | Services | SRV |
| 4601 | Road Transport | Services | SRV |
| 4701 | Rail Transport | Services | SRV |
| 4801 | Water, Pipeline and Other Transport | Services | SRV |
| 4901 | Air and Space Transport | Services | SRV |
| 5101 | Postal and Courier Pick-up and Delivery Service | Services | SRV |
| 5201 | Transport Support services and storage | Services | SRV |
| 5401 | Publishing (except Internet and Music Publishing) | Services | SRV |
| 5501 | Motion Picture and Sound Recording | Services | SRV |
| 5601 | Broadcasting (except Internet) | Services | SRV |
| 5701 | Internet Service Providers, Internet Publishing and Broadcasting, Websearch Portals and Data Processing | Services | SRV |
| 5801 | Telecommunication Services | Services | SRV |
| 6001 | Library and Other Information Services | Services | SRV |
| 6201 | Finance | Services | SRV |
| 6301 | Insurance and Superannuation Funds | Services | SRV |
| 6401 | Auxiliary Finance and Insurance Services | Services | SRV |
| 6601 | Rental and Hiring Services (except Real Estate) | Services | SRV |
| 6701 | Ownership of Dwellings | Dwellings | DWE |
| 6702 | Non-Residential Property Operators and Real Estate Services | Services | SRV |
| 6901 | Professional, Scientific and Technical Services | Services | SRV |
| 7001 | Computer Systems Design and Related Services | Services | SRV |
| 7210 | Employment, Travel Agency and Other Administrative Services | Services | SRV |
| 7310 | Building Cleaning, Pest Control and Other Support Services | Services | SRV |
| 7501 | Public Administration and Regulatory Services | Services | SRV |
| 7601 | Defence | Services | SRV |
| 7701 | Public Order and Safety | Services | SRV |
| 8010 | Primary and Secondary Education Services (incl Pre-Schools and Special Schools) | Services | SRV |
| 8110 | Technical, Vocational and Tertiary Education Services (incl undergraduate and postgraduate) | Services | SRV |
| 8210 | Arts, Sports, Adult and Other Education Services (incl community education) | Services | SRV |
| 8401 | Health Care Services | Services | SRV |
| 8601 | Residential Care and Social Assistance Services | Services | SRV |
| 8901 | Heritage, Creative and Performing Arts | Services | SRV |
| 9101 | Sports and Recreation | Services | SRV |
| 9201 | Gambling | Services | SRV |
| 9401 | Automotive Repair and Maintenance | Services | SRV |
| 9402 | Other Repair and Maintenance | Services | SRV |
| 9501 | Personal Services | Services | SRV |
| 9502 | Other Services | Services | SRV |

1. The views expressed in this paper are those of the authors and do not necessarily reflect those of The Australian Treasury or the Australian Government. [↑](#footnote-ref-2)
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3. In the US context, for example, OLG models have been developed and used to investigate the economic efficiency of privatising social security (Nishiyama and Smetters (2007)), the intergenerational effect of closing the fiscal gap (Nishiyama and Reichling (2015)), the macroeconomic effects of reforming the Internal Revenue Code (JCT, 2018), and the budgetary and economic effects of a wealth tax (He et al. (2019)). In the Australian context, for example, OLG models have been developed and used to estimate the aggregate and distributional effects of various policy proposals to reform the retirement income system (Kudrna and Woodland (2010)), the behavioural effects of means test on pensioners (Tran and Woodland (2011)), the fiscal effects of demographic change (Kudrna et al. (2015)), and the economic and welfare consequences of different budget repair measures (Kudrna and Tran (2018)). [↑](#footnote-ref-4)
4. For reference, see <https://www.dss.gov.au/seniors/benefits-payments/age-pension>. [↑](#footnote-ref-5)
5. Because efficiency units are perfect substitutes, we cannot identify the share of total hours employed by the representative firm in sector *j*. [↑](#footnote-ref-6)
6. See Ludwig (2007) and Heer and Maussner (2009). [↑](#footnote-ref-7)
7. Given superannuation is not explicitly modelled in this version of OLGA we group superannuation fund tax together with personal income tax and abstract from the concessional nature of taxation on contributions and earnings. [↑](#footnote-ref-8)
8. Source: https://www.humanservices.gov.au/individuals/enablers/income-test-pensions/30406 [↑](#footnote-ref-9)